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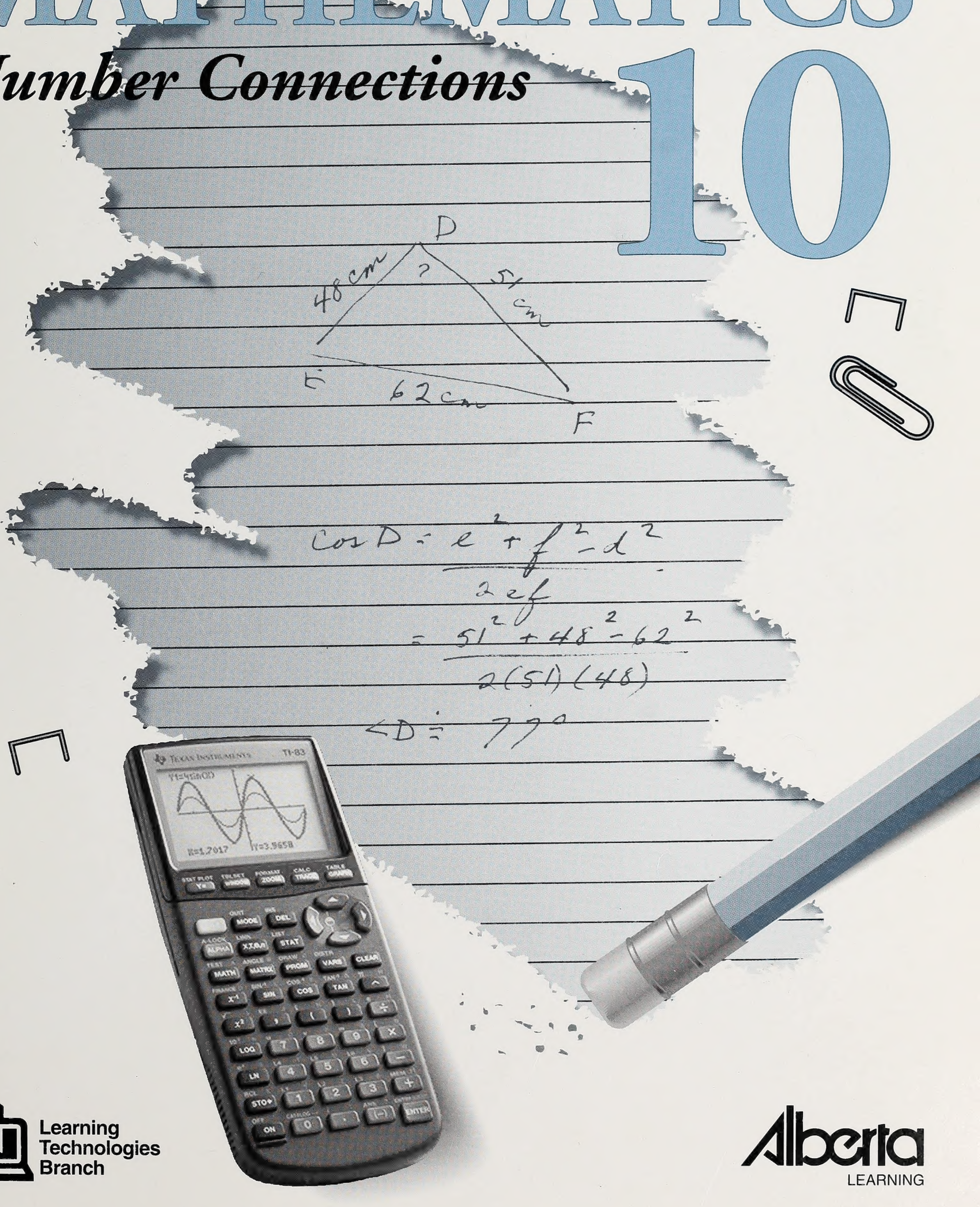
Module 4

MATHEMATICS 10

Number Connections


D
48 cm
?
51 cm
E
62 cm
F
$\cos D \doteq \frac{e^2+f^2-d^2}{2ef}$
$= \frac{51^2+48^2-62^2}{2(51)(48)}$
$\angle D \doteq 77°$
TEXAS INSTRUMENTS
TI-83

Learning Technologies Branch

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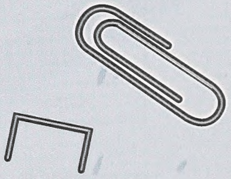
Pure MATHEMATICS 10

Module 4

Number Connections



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Pure Mathematics 10
Module 4: Number Connections
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2145-9

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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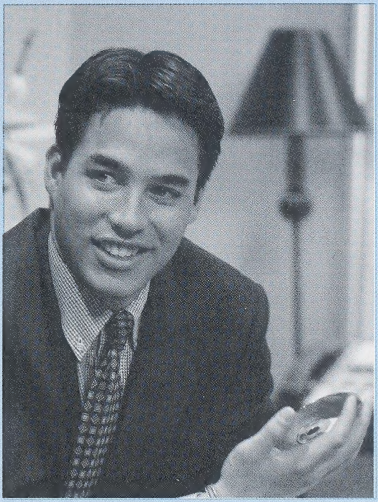
Welcome



Welcome to Module 4. We hope you'll enjoy your study of Number Connections.

Pure Mathematics 10

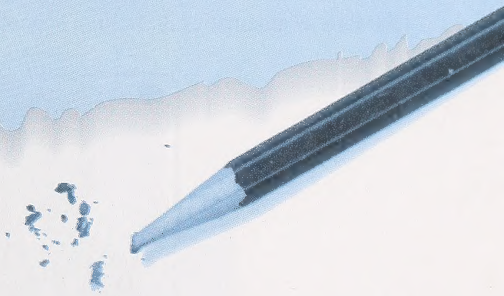
Module 1
Statistics and Data Tables

Module 2
Patterns and Relations

Module 3
Algebra

Module 4
Number Connections

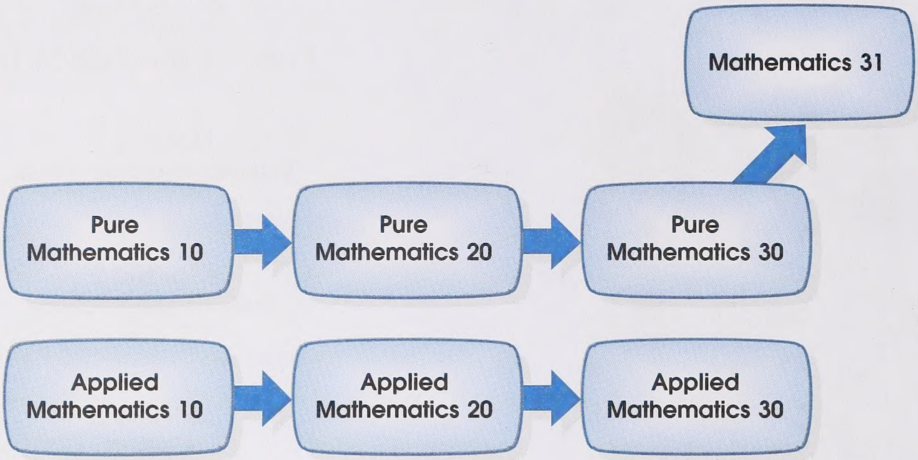
Module 5
Trigonometry



Pure Mathematics 10 contains five modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

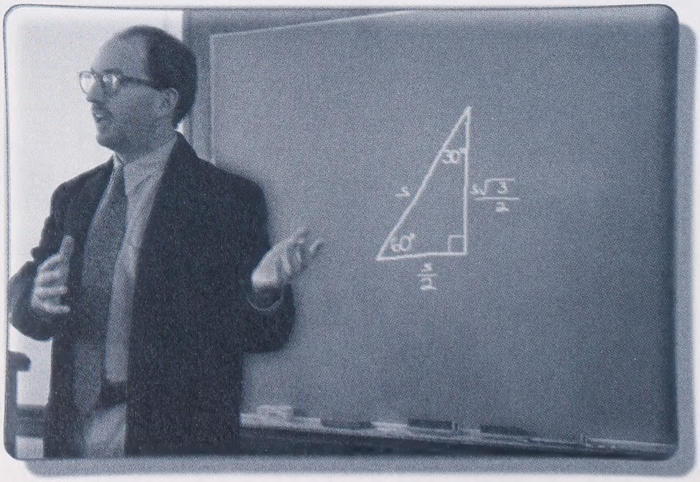
Pure Mathematics 10 is the first course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.



Can I Transfer to the Applied Mathematics Sequence of Courses?

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

Applied Topics

- linear programming
- data tables and trends
- design and layout
- metric and imperial measure
- data presentation
- vectors and matrices
- periodic, fractal, and recursive patterns
- financial decision making
- costing and design problems

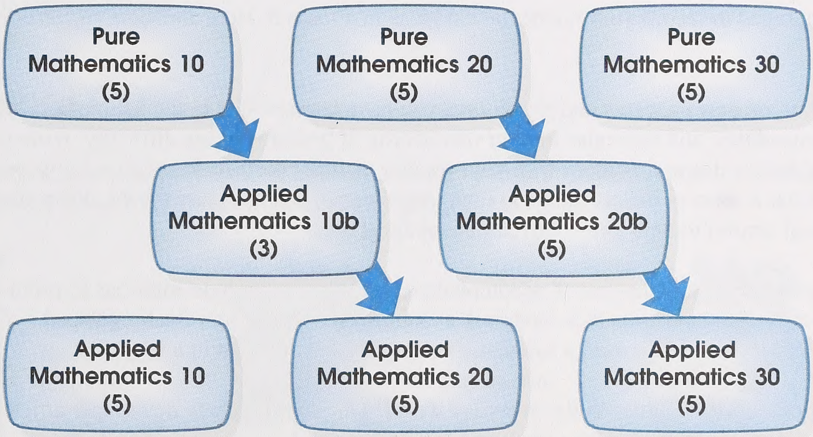
Common Topics

- spreadsheets
- line segments and linear graphs
- scaling
- triangles
- surveys
- financial mathematics
- quadratic functions
- circle geometry
- the bell curve

Pure Topics

- irrational numbers
- exponents
- polynomial and rational expressions
- mathematical expectation
- growth patterns
- linear and non-linear systems
- operations on functions
- mathematical reasoning
- exponential and logarithmic functions
- conics
- combinations
- trigonometric functions

If you want to transfer from the Pure Mathematics 10–20–30 sequence to the Applied Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to the Applied Mathematics sequence of courses after successfully completing Pure Mathematics 10, you may take the 3-credit course called Applied Mathematics 10b or you may take the 5-credit course called Applied Mathematics 10. If you decide to transfer after successfully completing Pure Mathematics 20, you will take the 5-credit course called Applied Mathematics 20b. The two bridging courses are shown in the following diagram:


Pure Mathematics 10 (5)
Pure Mathematics 20 (5)
Pure Mathematics 30 (5)
Applied Mathematics 10b (3)
Applied Mathematics 20b (5)
Applied Mathematics 10 (5)
Applied Mathematics 20 (5)
Applied Mathematics 30 (5)

Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

This icon shows which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.

- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an "aha!") or when you first used a calculator or spreadsheet program in a new way.

- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 10 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Centre:

- the *MATHPOWER™ 10* textbook and data disks, Western Edition, published by McGraw-Hill Ryerson (1998)

- a graphing calculator
  Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* Algebra FX-2.0, CFX-9850 GA-Plus, CFX-9850 G, CFX-9800 G, or FX-9700 series; *Sharp* EL-9600C, EL-9600, EL-9200, or EL-9300; or *Hewlett-Packard* HP 39g (only approved for 2001–2003 school years). **Note:** Some calculators are no longer commercially manufactured.

- spreadsheet program
  Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh or for Windows, *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

Note: Examples in this course show the TI-83 graphing calculator and the *ClarisWorks™ 4.0* spreadsheet program where applicable.

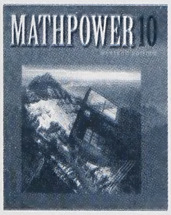
You will need access to a computer (mandatory) and a videocassette player (optional).

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a binder in which to respond to the questions asked
- a binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.



Refer to the textbook.

View a videocassette.

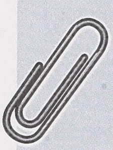
Use the Internet to explore a topic.

Answer the questions in the Assignment Booklet.

Use the Computer Data Bank.

Remember: *Any Internet website address given in this module is subject to change.*

CONTENTS



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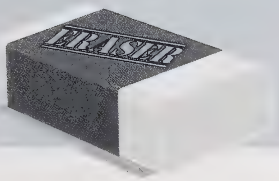
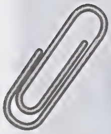
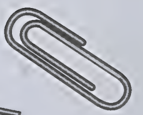




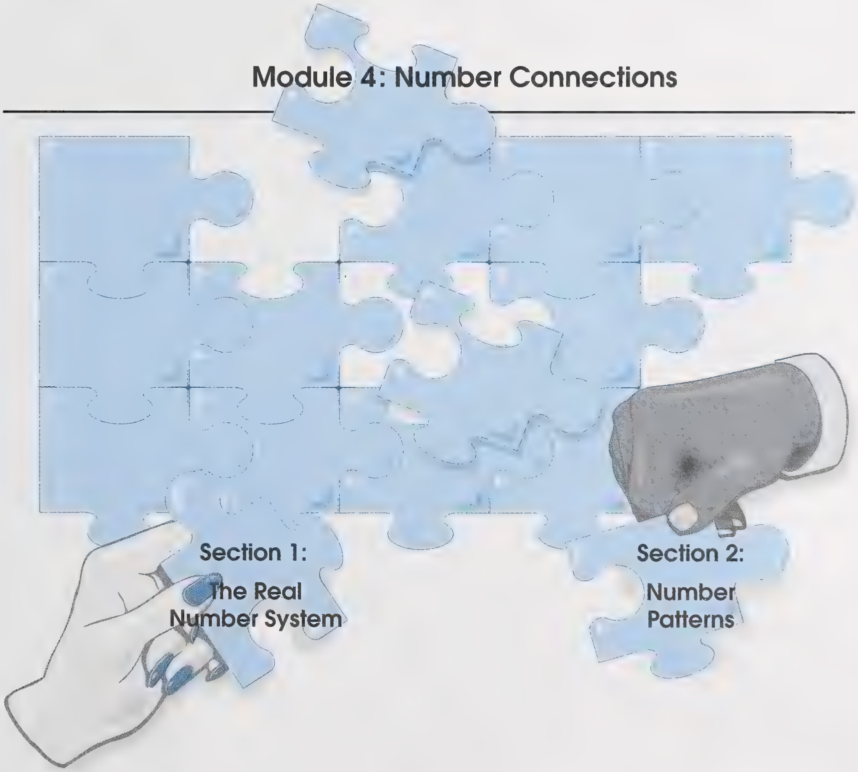


Module Overview

Do you like putting together jigsaw puzzles? A large jigsaw puzzle contains many interconnecting pieces and can be quite challenging to complete. But if you pay attention to the various patterns and colours, you can fit the pieces together.

Similarly, the study of mathematics is easier if you connect your previous knowledge to the new concepts you encounter. You will discover that the pieces of the mathematical puzzle are beginning to fit together, thus giving you a sense of the big picture.

In this module, you will investigate number connections. In Section 1, you will examine the real number system, perform operations on irrational numbers, and work with powers that have rational exponents. In Section 2, you will explore the patterns in number sequences.



Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Number Connections, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

In this module, you will be directed to the accompanying Assignment Booklets. You are expected to complete two Assignment Booklets for this module. Your grading in this module is based in part upon the assignments you submit for evaluation. The mark distribution is as follows:

Assignment Booklet 4A	
Section 1 Assignment	**50 marks**
Assignment Booklet 4B	
Section 2 Assignment	**35 marks**
Final Module Assignment	**15 marks**
TOTAL	**100 marks**

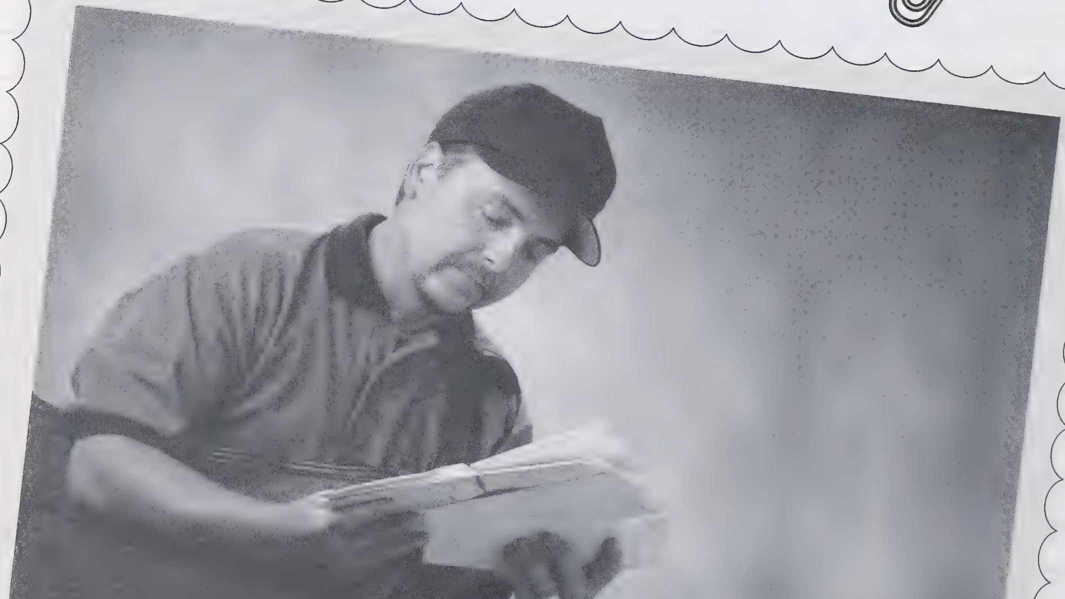
Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklet, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 10.

Good luck!

Section 1

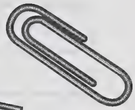


The Real Number System

Canada has a national postal system. Each province and territory has an education system. Your body is made up of several systems—the circulatory, muscular, skeletal, reproductive, nervous, and digestive systems. Your house has a heating system; it may even have an air-conditioning system.

There are also systems in mathematics. Up to this point, you have examined natural numbers, integers, and rational numbers—each a part of the real number system. What does the real number system have in common with the postal system, the education system, or the nervous system? All systems have interdependent parts that regularly interact to form a unified whole.

In this section, you will investigate irrational numbers, which together with the rational numbers forms the real number system. You will discover that an irrational number can be written in different but equivalent forms. You will also perform operations on irrational numbers. In addition, you will review strategies for problem solving, read and give instructions, and discover math connections.



Activity 1: Irrational Numbers

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

People seem to be continually sorting things. A clerk places different denominations of coins and paper currency in different compartments of the drawer of a cash register. Scientists sort living things into two kingdoms—the animal kingdom and the plant kingdom—and then subdivide each kingdom into subsets.

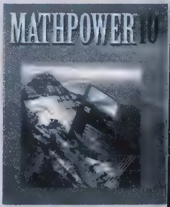
Mathematicians sort decimal numbers into three groups:

- terminating decimal numbers
- repeating decimal numbers
- non-terminating and non-repeating decimal numbers

You will now use a calculator to investigate the relationship between repeating decimal numbers and fractions.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization



Turn to “TECHNOLOGY” on page 4 of *MATHPOWER™ 10* and read “Calculators and Repeating Decimals.”

real numbers: the set of all numbers on the number line

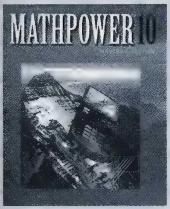
irrational number: a real number that cannot be expressed as a ratio of two integers, a terminating decimal number, or a repeating decimal number

1. Answer the following questions on page 4 of the textbook:

   a. questions 1 and 2 of Investigation 1, "One Digit Repeating"
   b. questions 1 and 2 of Investigation 2, "Two Digits Repeating"
   c. question 1 of Investigation 3, "Non-Repeating Parts"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Terminating decimals, repeating decimals, and non-terminating and non-repeating decimals are all **real numbers**.



Turn to page 7 of *MATHPOWER™ 10*. Read from the red line below Example 1 to Example 2. Then work through Example 2.

2. Answer the following questions on page 8 of the textbook:

   a. questions 21 to 26 of "Practice"
   b. questions 27 to 30 of "Practice"
   c. questions 49 to 53 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 1.

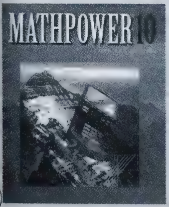
---

rational number: a real number that can be expressed as a ratio of two integers, a terminating decimal number, or a repeating decimal number

Real numbers were first described by a German mathematician named Richard Dedekind (1831–1916). He separated real numbers into two groups: **rational numbers** and **irrational numbers**.

If you have access to the Internet and wish to learn more about Richard Dedekind, use one of the Internet's search engines to find more information or view the following websites:

http://history.math.csusb.edu/Mathematicians/Dedekind.html
http://www.astro.virginia.edu/~eww6n/bios/Dedekind.html

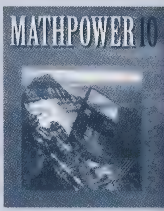


Turn to page 5 of *MATHPOWER™ 10* and read "The Real Number System."

3. Answer "Explore: Complete the Table" on page 5 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 1.

Now, turn to pages 6 and 7 of *MATHPOWER™ 10* and read from the red line on page 6 to the bottom of page 6. Then work through Example 1 on page 7.

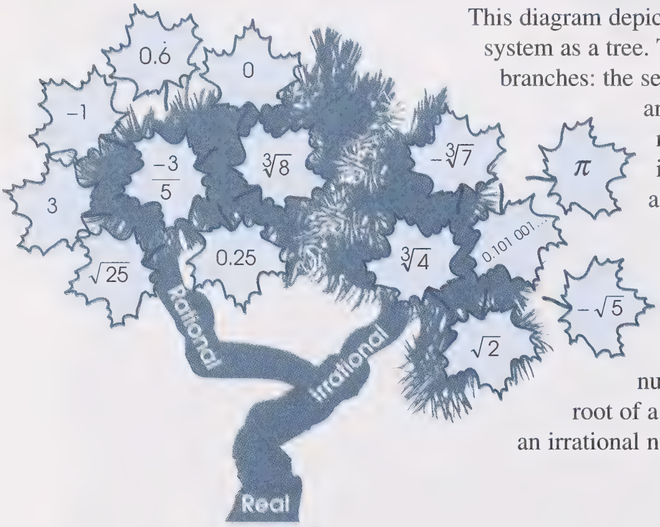


4. Answer the following questions on page 8 of the textbook:

- **a.** questions 1 to 3 of "Practice"
- **b.** questions 4 to 15 of "Practice"
- **c.** questions 16 to 19 of "Practice"
- **d.** question 56 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Square Roots



This diagram depicts the real number system as a tree. The tree has two main branches: the set of rational numbers and the set of irrational numbers. The numbers in these sets are shown as leaves on the tree.

As you can see from the diagram, the square root of a perfect square is a rational number and the square root of a non-perfect square is an irrational number.

Mathematical Process

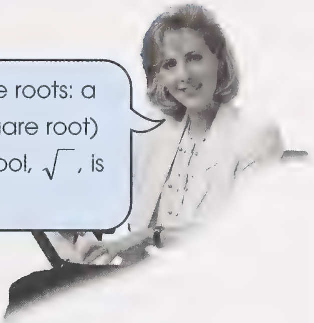
☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☒ Visualization

You can use mental math or a calculator to find the exact value of the square root of a perfect square. You can use a calculator to find the approximate decimal value of the square root of a non-perfect square.

It is easy to make a calculation error when you are solving problems. Therefore, to decide if an answer is reasonable, you should estimate the answer and compare the calculated answer to the estimate.

Mathematical Process

☐ Communication
☐ Connection
☒ Estimation
☒ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization



Remember, all positive numbers have two square roots: a positive square root (also called the principal square root) and a negative square root. The square root symbol, $\sqrt{}$, is used to denote the positive square root.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Example

Estimate the value of $\sqrt{184}$. Then use a calculator to find the decimal approximation. Round your answer to the nearest tenth.

Solution

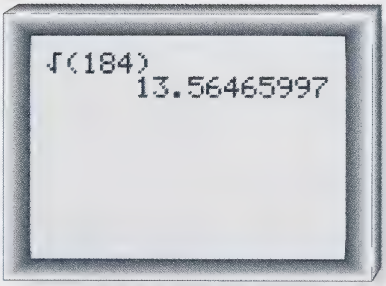
Step 1: Estimate the value of $\sqrt{184}$.

$$\sqrt{100} = 10 \qquad \sqrt{225} = 15$$

Because 184 is between the perfect squares of 100 and 225, the value of $\sqrt{184}$ is between 10 and 15.

Step 2: Press the following sequence of keys to calculate the approximate decimal value of $\sqrt{184}$.

2nd [√‾] 1 8 4) ENTER



```
√(184)
          13.56465997
```

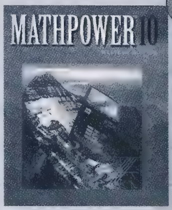
Remember to enter the closing parenthesis around the number under the square root sign.

$\therefore \quad \sqrt{184} \doteq 13.6$

Step 3: Decide if the answer is reasonable by comparing the calculated answer to the estimate.

Because 13.6 is between 10 and 15, the answer is reasonable.

Turn to pages 13 and 14 of *MATHPOWER™ 10*. Read from the red line below Example 1 to Example 2. Then work through Examples 2 and 3.



5. Answer the following questions on page 14 of the textbook:

- **a.** question 1 of "Practice"
- **b.** questions 2, 4, 6, 8, 10, and 12 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Now, you will use the rules for order of operations to find the approximate value of an irrational expression. Work through the following example.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☒ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Example

Estimate the value of $5\sqrt{3}-2\sqrt{7}$. Then find an approximate decimal value, rounded to the nearest tenth.

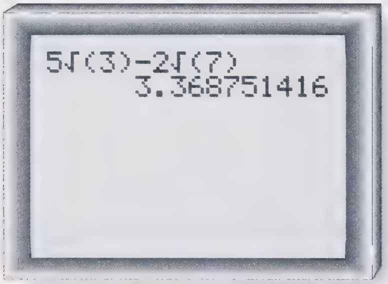
Solution

Step 1: Estimate the value.

$$\begin{aligned}5\sqrt{3}-2\sqrt{7} &\doteq 5\times2-2\times3\\ &\doteq 10-6\\ &\doteq 4\end{aligned}$$

Step 2: Press the following key sequence to calculate the approximate decimal value of $5\sqrt{3}-2\sqrt{7}$.

5 2nd [√] 3) – 2 2nd [√] 7) ENTER



```
5√(3)-2√(7)
           3.368751416
```

Remember to enter the closing parenthesis around the number under the square root sign.

$\therefore\ 5\sqrt{3}-2\sqrt{7}\doteq 3.4$

Step 3: Decide if the calculated value is reasonable by comparing the estimate to the calculated value.

Because 3.4 is close to the estimate of 4, the calculated value is reasonable.

6. Turn to pages 14 and 15 of *MATHPOWER™ 10* and answer the following:

a. questions 15, 17, 19, and 21 of "Practice"
b. questions 23, 25, and 27 of "Practice"

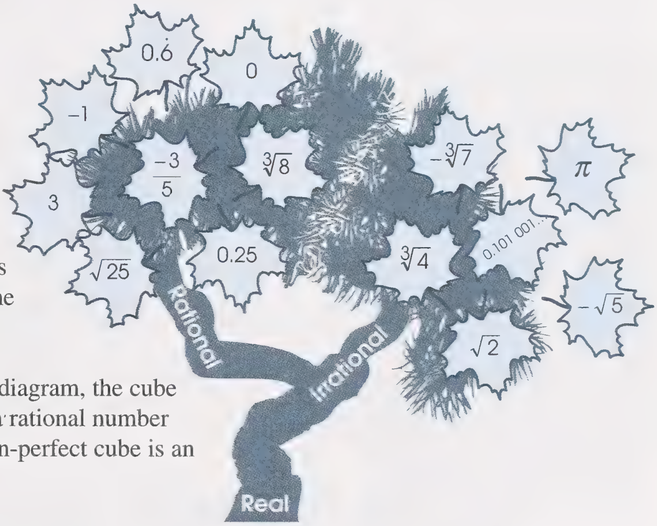
Compare your responses with those in the Appendix, Section 1: Activity 1.

Cube Roots

Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ☒ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization

This diagram depicts the real number system as a tree. The tree has two main branches: the set of rational numbers and the set of irrational numbers. The numbers in these sets are shown as leaves on the tree.

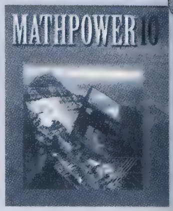


As you can see from the diagram, the cube root of a perfect cube is a rational number and the cube root of a non-perfect cube is an irrational number.

You can use mental math to find the exact value of the cube root of a perfect cube.

Remember, all positive numbers have one cube root: a positive root. For example, because $5 \times 5 \times 5 = 125$, the cube root of 125 is 5. Likewise, all negative numbers have one cube root—a negative root. For example, because $(-5) \times (-5) \times (-5) = -125$, the cube root of −125 is −5.

7. Turn to page 15 of *MATHPOWER™ 10* and do questions 34 to 38 of “Practice.”



Compare your responses with those in the Appendix, Section 1: Activity 1.

You can use a calculator to find the approximate decimal value of the cube root of a non-perfect cube.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Example

Estimate the value of $\sqrt[3]{70}$. Then use a calculator to find the decimal approximation, rounded to the nearest tenth.

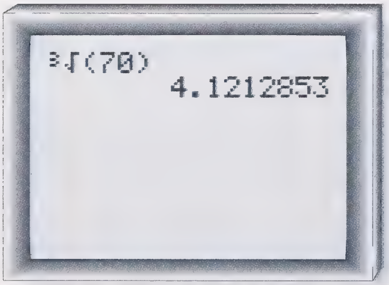
Solution

Step 1: Estimate the value of $\sqrt[3]{70}$.

Because $4\times4\times4=64$ and $5\times5\times5=125$, the value of $\sqrt[3]{70}\doteq4$.

Step 2: Calculate the approximate decimal value and round to the nearest tenth.

First, press MATH on your graphing calculator. This action displays the MATH menu. Then press 4, or alternatively, use the arrow keys to move the cursor to “4:$\sqrt[3]{\ }$(” and press ENTER. Either action pastes “$\sqrt[3]{\ }$(” onto the screen. Finally, press 7 0) ENTER.



```
³√(70)
              4.1212853
```

Remember to enter the closing parenthesis around the number under the cube root sign.

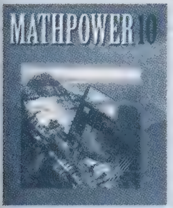
$\therefore\ \sqrt[3]{70}\doteq4.1$

8. Turn to page 15 of *MATHPOWER™ 10* and do questions 39 to 44 of “Practice.”

Compare your responses with those in the Appendix, Section 1: Activity 1.

Formulas

To solve some problems, you need to use a formula. Sometimes the formula is given. At other times, you are expected to know the formula.



Turn to page 13 of *MATHPOWER™ 10* and work through Example 1.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

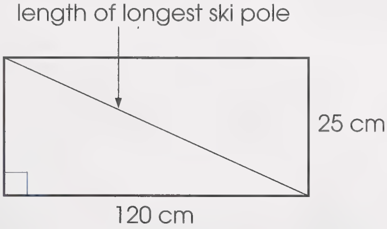
Now, work through the following example.

Example

A box is 120 cm long and 25 cm wide. What is the length of the longest ski pole that could be packed to lie flat in the box?

Solution

Step 1: Make a diagram to help you understand the problem.



The length of the longest ski pole is equivalent to the diagonal of the rectangular bottom of the box. This diagonal is the hypotenuse of a right triangle. Therefore, you can use the Pythagorean Theorem (also called the Pythagorean Relation) to calculate this length.

Step 2: Write the formula for the Pythagorean Theorem and substitute the given values. Simplify the equation.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 25^2 + 120^2 &= c^2 \\ 625 + 14\,400 &= c^2 \\ 15\,025 &= c^2 \end{aligned}$$

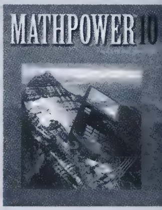
Step 3: Solve for c.

Ask yourself, "What is the square root of 15 025?" Since a measurement is required, give only the positive square root of 15 025.

$$\begin{aligned} c^2 &= 15\,025 \\ c &\doteq 122.6 \end{aligned}$$

Therefore, the length of the longest ski pole that could be packed is about 122.6 cm.

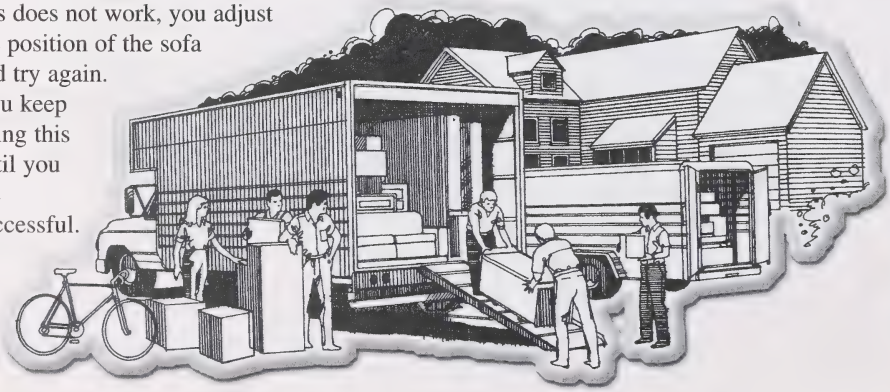
9. Turn to pages 15 and 16 of *MATHPOWER™ 10* and answer questions 48, 49, 51, 52, and 54.a. of "Applications and Problem Solving."



Compare your responses with those in the Appendix, Section 1: Activity 1.

Now Try This

Moving a large piece of furniture, such as a sofa, through a doorway can pose quite a problem. To solve this problem, you probably make a guess as how to position the sofa so it will go through the doorway. If this does not work, you adjust the position of the sofa and try again. You keep doing this until you are successful.



Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ☒ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

You can also use the guess and check strategy to solve mathematical problems. Turn to "PROBLEM SOLVING" on page 10 of *MATHPOWER™ 10* and read "Guess and Check."

10. Answer questions 7 and 9 of "Applications and Problem Solving" on page 11.

Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

When you ask questions in mathematics, it is important that your language is clear.

Consider the following question:

What is two times five plus three?

This question can be interpreted two ways:

- $2\times5+3$
- $2\times(5+3)$

To be clear, punctuation marks (or pauses in spoken language) are needed.

- What is two times five, plus three?
- What is two times, five plus three?

Different wording could also be used to make the meaning clearer.

- What is three more than the product of two and five?
- What is two times the sum of five and three?

11. Write each of the following in words. Be sure the meaning is clear.

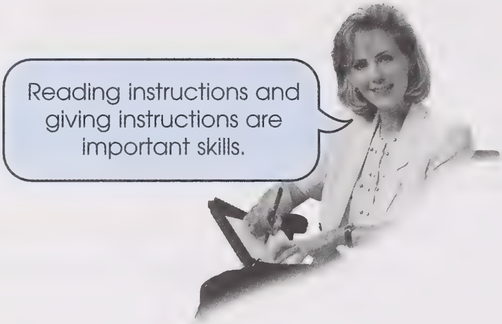
a. $1+6\div3$ **b.** $8-2\times3$ **c.** $14-(1+2)$

Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process

☒ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☒ Technology
☐ Visualization

12. a. Turn to the manual that accompanies your graphing calculator and find out how to enter a negative number. If you are using the TI-83, see "negation" in the index of the manual. Then write the directions, including diagrams, of how to use a graphing calculator to evaluate the following expression:


Reading instructions and giving instructions are important skills.

$$(-13)\times(-15)-(-25)$$

b. Turn to the manual that accompanies your graphing calculator and find out how to display an answer in fraction form. If you are using the TI-83, see "Frac (to fraction)" in the index of the manual. Then write the directions, including diagrams, of how to use a graphing calculator to evaluate the following expression and express the answer as a fraction:

$$\frac{8}{25}\times\frac{5}{32}$$

c. Turn to the manual that accompanies your graphing calculator and find out how to evaluate a power. If you are using the TI-83, see "power (^)" in the index of the manual. Then write the directions, including diagrams, of how to use a graphing calculator to evaluate the following expression:

$$14^5$$

Compare your responses with those in the Appendix, Section 1: Activity 1.

Looking Back

In this activity, you reviewed the relationship between the sets of numbers that make up the real number system. You worked with decimal approximations of irrational numbers, used the guess and check problem-solving strategy, and gave instructions for various mathematical processes.

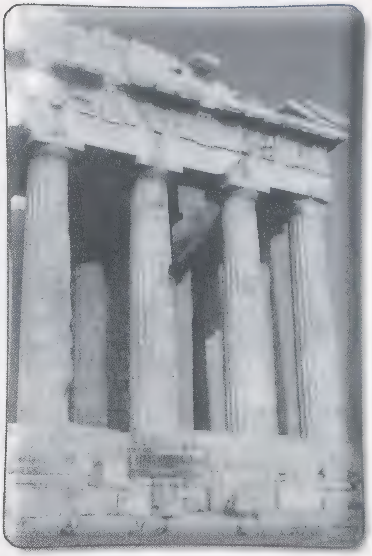
In your journal, draw a diagram to demonstrate the relationship between the subsets of numbers within the real number system. Then write a definition for each of the subsets and include examples.

Mathematical Process

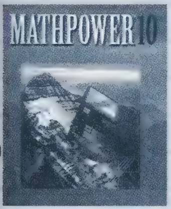
☒ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

Activity 2: Radicals

Over two thousand years ago in ancient Greece, a group of people set up a club to study mathematics. The members of this group were called Pythagoreans, after their leader Pythagoras. The Pythagoreans had ideas about numbers, which they wanted to keep secret. According to legend, a member was killed because he told a non-member a proof showing that $\sqrt{2}$ is not a rational number.



Even though $\sqrt{2}$ is not a rational number, the Pythagoreans knew that this number existed. After all, they had calculated this length using the Pythagorean Theorem (also called the Pythagorean Relation).



1. Turn to "GETTING STARTED" on page 2 of *MATHPOWER™ 10* and answer question 1 of "Using the Pythagorean Theorem."

Compare your responses with those in the Appendix, Section 1: Activity 2.

An expression of the form $\sqrt[n]{x}$, where n is a natural number, is called a **radical**; n is called the **index**, and x is called the **radicand**. The following are examples of radicals:

radical: a number that involves a root sign

index: the number in the upper left-hand corner of the root sign that indicates which root is to be extracted

radicand: the number or algebraic expression under the root sign

$\sqrt[4]{7}$ $\sqrt{5}$ $\sqrt[3]{28}$

In the expression $\sqrt{5}$, the index is understood to be 2.

In Activity 1, you approximated the value of radical expressions, such as $3\sqrt{2}\times 5\sqrt{18}$. In this activity, you will simplify radical expressions rather than use decimal approximations.

2. Turn to page 17 of *MATHPOWER™ 10* and answer "Explore: Complete the Table."

Compare your responses with those in the Appendix, Section 1: Activity 2.

You have discovered the following properties about multiplying and dividing radicals.

- The square root of the product of two numbers equals the product of the square roots of the numbers. This property can be generalized as $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$, where $a \geq 0$ and $b \geq 0$.
- The square root of the quotient of two numbers equals the quotient of the square roots of the numbers. This property can be generalized as $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$.

To discover how these properties are used to simplify radicals, work through the following example.

Example

Simplify the following radicals:

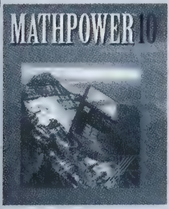
a. $\sqrt{48}$

b. $\sqrt{\frac{6}{25}}$

Solution

a.
$$\begin{aligned}\sqrt{48} &= \sqrt{16\times 3}\\ &= \sqrt{16}\times\sqrt{3}\\ &= 4\sqrt{3}\end{aligned}$$

b.
$$\begin{aligned}\sqrt{\frac{6}{25}} &= \frac{\sqrt{6}}{\sqrt{25}}\\ &= \frac{\sqrt{6}}{5}\end{aligned}$$



3. Turn to page 19 of *MATHPOWER™ 10* and answer questions 1, 3, 5, 7, 9, 11, 13, and 15 of "Practice."

4. Simplify the following radicals:

a. $\sqrt{\frac{1}{4}}$ **b.** $\sqrt{\frac{5}{36}}$ **c.** $\sqrt{\frac{7}{16}}$ **d.** $\sqrt{\frac{2}{9}}$

Compare your responses with those in the Appendix, Section 1: Activity 2.

Operations on Radicals

You will now simplify radical expressions by performing operations on the radicals in each expression and expressing the answer as a radical in simplest form.

The following properties are used to multiply and divide radicals:

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, where $a \geq 0$ and $b \geq 0$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, where $a \geq 0$ and $b > 0$

Work through the following example.

Example

Simplify the following expressions:

a. $\sqrt{3} \times \sqrt{6}$ **b.** $\frac{\sqrt{14}}{\sqrt{2}}$

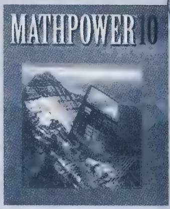
Solution

a.
$$\begin{aligned}\sqrt{3} \times \sqrt{6} &= \sqrt{3 \times 6}\\ &= \sqrt{18}\\ &= \sqrt{9 \times 2}\\ &= 3\sqrt{2}\end{aligned}$$

b.
$$\begin{aligned}\frac{\sqrt{14}}{\sqrt{2}} &= \sqrt{\frac{14}{2}}\\ &= \sqrt{7}\end{aligned}$$

5. Turn to page 19 of *MATHPOWER™ 10* and answer the following:

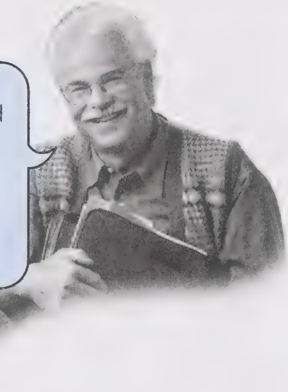
- **a.** questions 34 to 37 of "Practice"
- **b.** questions 16 to 20 of "Practice"



Compare your responses with those in the Appendix, Section 1: Activity 2.

entire radical: a number written entirely underneath a root sign

mixed radical: a number written with one part underneath a root sign and the other part outside the root sign



Numbers like $\sqrt{18}$ and $\sqrt{\frac{2}{9}}$ are called **entire radicals** or **pure radicals**. Numbers like $3\sqrt{2}$ and $\frac{\sqrt{2}}{3}$ are called **mixed radicals**.

To multiply monomials, you multiply the numerical factors and then you multiply the variable factors. To divide monomials, you divide the numerical coefficients and then divide the variables.

$$3a\left(5a^3\right)=15a^4 \qquad \frac{8ab}{4a}=2b$$

When you multiply and divide mixed radicals, use a process similar to multiplying monomials in algebra.

$$3\sqrt{2}\times 2\sqrt{5}=6\sqrt{10} \qquad \frac{8\sqrt{6}}{4\sqrt{2}}=2\sqrt{3}$$

Mathematical Process

- ▪ Communication
- ☒ Connection
- ▪ Estimation
- ▪ Mental Math
- ▪ Problem Solving
- ▪ Reasoning
- ▪ Technology
- ▪ Visualization

Turn to page 18 of *MATHPOWER™ 10* and work through Example 3.

6. Answer the following questions on pages 19 and 20 of the textbook:

- **a.** questions 38, 41, and 42 of "Practice"
- **b.** questions 22 to 24 of "Practice"
- **c.** questions 56 and 60 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process

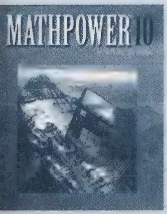
☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

When you add and subtract monomials, you combine like terms.

$$3a + 5a = 8a \qquad a^2 - ab + 3ab - b^2 = a^2 + 2ab - b^2$$

You use a similar process when you add and subtract radicals.

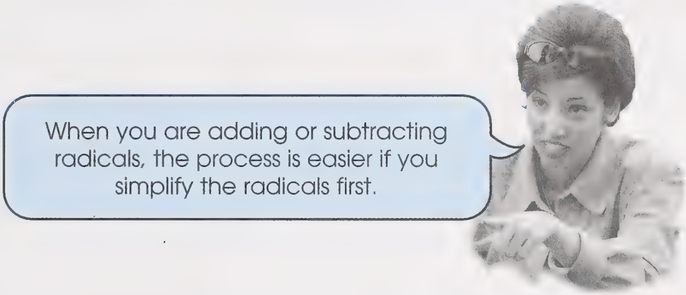
$$3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2} \qquad \begin{aligned} 2\sqrt{3} - 5\sqrt{6} + 4\sqrt{3} &= 2\sqrt{3} + 4\sqrt{3} - 5\sqrt{6} \\ &= 6\sqrt{3} - 5\sqrt{6} \end{aligned}$$



7. Turn to pages 23 and 24 of *MATHPOWER™ 10* and answer the following:

- **a.** questions 1, 3, 5, and 7 of "Practice"
- **b.** questions 8, 10, 12, and 14 of "Practice"
- **c.** question 71 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.


When you are adding or subtracting radicals, the process is easier if you simplify the radicals first.

Turn to page 21 of *MATHPOWER™ 10* and work through Example 1.

8. Answer the following questions on pages 23 and 24 of the textbook:

- **a.** questions 15, 17, 19, and 21 of "Practice"
- **b.** questions 22, 24, 26, and 28 of "Practice"
- **c.** question 66 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Binomial Radicals

Mathematical Process

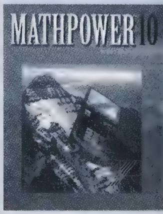
- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

In algebra, the distributive property is used to multiply a binomial by a monomial.

$$\begin{aligned} 3a(2a+b) &= 3a(2a+b) \\ &= 6a^2+3ab \end{aligned}$$

The distributive property is also used to multiply a binomial radical by a monomial radical.

Turn to page 22 of *MATHPOWER™ 10* and work through Example 2.



9. Answer the following questions on pages 23 and 24 of the textbook:

a. questions 29 to 34 of “Practice”
b. question 64 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process

- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ☒ Visualization

When multiplying two binomials in algebra, the mnemonic FOIL helps you remember that the **F**irst, **O**utside, **I**nside, and **L**ast terms are multiplied.

$$\begin{aligned} (x+3)(x-1) &= \underbrace{x(x)}_{\text{F}}+\underbrace{x(-1)}_{\text{O}}+\underbrace{3(x)}_{\text{I}}+\underbrace{3(-1)}_{\text{L}} \\ &= x^2-x+3x-3 \\ &= x^2+2x-3 \end{aligned}$$

FOIL can also be used to multiply two binomial radicals.

$$\begin{aligned} \left(\sqrt{2}+3\right)\left(\sqrt{2}-1\right) &= \underbrace{\sqrt{2}\left(\sqrt{2}\right)}_{\text{F}}+\underbrace{\sqrt{2}(-1)}_{\text{O}}+\underbrace{3\left(\sqrt{2}\right)}_{\text{I}}+\underbrace{3(-1)}_{\text{L}} \\ &= 2-\sqrt{2}+3\sqrt{2}-3 \\ &= 2\sqrt{2}-1 \end{aligned}$$

Turn to page 22 of *MATHPOWER™ 10* and work through Example 3.

10. Answer the following questions on pages 23 and 24 of the textbook:

- **a.** questions 35 to 39 of "Practice"
- **b.** question 65 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

A pair of binomials that are the same except that the sign in the second binomial is opposite the first are called **conjugate binomials.**

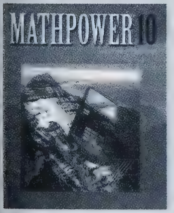
conjugate binomials: a pair of binomials that are the same except that the sign in the second is the opposite of the first

In algebra, the product of conjugate binomials is always a binomial.

$$(x+3)(x-3)=x^2-9$$

The product of conjugate binomial radicals is always a rational number.

$$\begin{aligned}\left(\sqrt{2}+3\right)\left(\sqrt{2}-3\right)&=2-9\\&=-7\end{aligned}$$



Turn to page 22 of *MATHPOWER™ 10* work through Example 4.

11. Answer the following questions on pages 23 and 24 of the textbook:

- **a.** questions 40 to 42 of "Practice"
- **b.** question 67 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Fractions with Radicals

Fractions are usually written in simplest form. For example, $\frac{6}{8} = \frac{3}{4}$.

To simplify a fraction with a radical in the denominator, multiply the numerator and the denominator by the radical. This process is called **rationalizing the denominator** because it produces a rational number in the denominator.

rationalizing the denominator: the process of eliminating a radical from the denominator of a fraction

$$\begin{aligned} \frac{1}{\sqrt{2}} &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

Turn to page 18 of *MATHPOWER*™ *10* and work through Example 1.c. Then work through Example 4 on page 19.

12. Answer the following questions on page 20 of the textbook:

a. questions 46, 48, 50, 52, and 54 of "Practice"
b. question 61 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Conjugate binomials can be used to simplify a fraction with a binomial radical in the denominator.

Turn to page 22 of *MATHPOWER*™ *10* and work through Example 5.

13. Answer the following questions on pages 23 and 24 of the textbook:

a. questions 44, 46, 48, 50, and 52 of "Practice"
b. questions 68 and 69 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process

☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

When rational numbers written in the form $\frac{a}{b}$ are simplified, the denominator is a natural number.

$$\frac{5}{-8}=\frac{-5}{8} \text{ or } -\frac{5}{8}$$

When you simplify fractions with radicals, you should also ensure that the denominator is a natural number.

Example

Simplify each of the following:

a. $\frac{3}{\sqrt{5}+3}$ **b.** $\frac{2}{\sqrt{3}-3}$

Solution

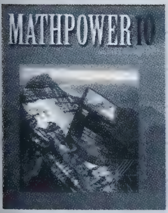
a.

$$\begin{aligned}\frac{3}{\sqrt{5}+3}&=\frac{3}{\sqrt{5}+3}\times\frac{\sqrt{5}-3}{\sqrt{5}-3}\\&=\frac{3\sqrt{5}-9}{5-9}\\&=\frac{3\sqrt{5}-9}{-4}\\&=\frac{3\sqrt{5}-9}{-4}\times\frac{-1}{-1}\\&=\frac{-3\sqrt{5}+9}{4}\\&=\frac{9-3\sqrt{5}}{4}\end{aligned}$$

b.

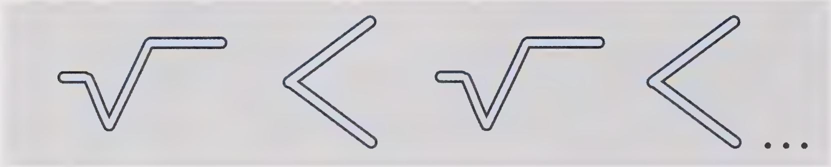
$$\begin{aligned}\frac{2}{\sqrt{3}-3}&=\frac{2}{\sqrt{3}-3}\times\frac{\sqrt{3}+3}{\sqrt{3}+3}\\&=\frac{2\left(\sqrt{3}+3\right)}{3-9}\\&=\frac{\overset{1}{\cancel{2}}\left(\sqrt{3}+3\right)}{-\underset{3}{\cancel{6}}}\\&=\frac{\sqrt{3}+3}{-3}\times\frac{-1}{-1}\\&=\frac{-\sqrt{3}-3}{3}\end{aligned}$$

14. Turn to page 23 of *MATHPOWER*™ *10* and do questions 43 and 45 of "Practice."



Compare your responses with those in the Appendix, Section 1: Activity 2.

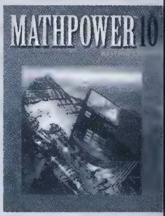
Comparing and Ordering Radical Expressions



Mathematical Process
- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

To compare fractions, you generally use equivalent fractions with a common denominator. For example, $\frac{3}{8} > \frac{1}{3}$ because $\frac{3}{8} = \frac{9}{24}$, $\frac{1}{3} = \frac{8}{24}$, and $\frac{9}{24} > \frac{8}{24}$.

To compare mixed radicals, you generally write the mixed radicals as entire radicals.



Turn to page 18 of *MATHPOWER™ 10* and work through Example 2.

15. Answer the following questions on page 19 of the textbook:

- **a.** questions 25, 27, and 29 of "Practice"
- **b.** questions 31 to 33 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 2.

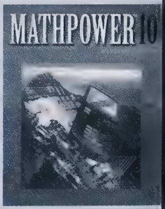
Mathematical Process
- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

To compare fractions, you sometimes express the fractions as decimal numbers. For example, $\frac{3}{8} > \frac{1}{3}$ because $\frac{3}{8} = 0.375$ and $\frac{1}{3} = 0.333\ldots$.

To compare radical expressions with more than one term, you can use the decimal approximation. For example, $1+\sqrt{3} < \sqrt{15}-1$ because $1+\sqrt{3} \doteq 2.732$, $\sqrt{15}-1 \doteq 2.873$, and 2.73 < 2.873.

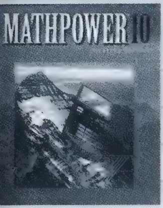
16. Arrange the following radical expressions in order from least to greatest:

$$\sqrt{7}+2 \qquad \sqrt{3}+3 \qquad \sqrt{26}-1 \qquad 6-\sqrt{3}$$



17. Turn to pages 23 and 24 of *MATHPOWER™ 10* and answer questions 61 and 62 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 1: Activity 2.



Now Try This

18. Answer the following problems in *MATHPOWER™ 10*:

a. "LOGIC POWER" on page 9
b. "PATTERN POWER" on page 16
c. "LOGIC POWER" on page 24

Compare your responses with those in the Appendix, Section 1: Activity 2.

Looking Back

Mathematical Process
☒ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

In this activity you worked with and simplified radicals. You performed operations on radicals and binomial radicals. You also compared radicals and used logic to solve problems.

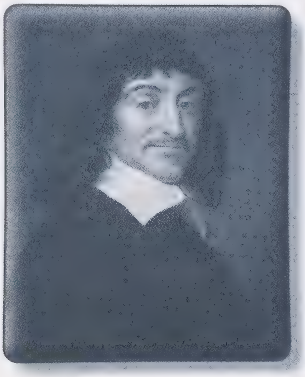
In your journal, write the three properties a radical must have in order to be in simplest form. Explain how to tell the difference between like radicals and unlike radicals.

Activity 3: Rational Exponents

Over the centuries, mathematicians have recognized the need for a symbol to represent an exponent. The following table shows a few ways that the powers $7x^2$ and $5x^3$ were written:

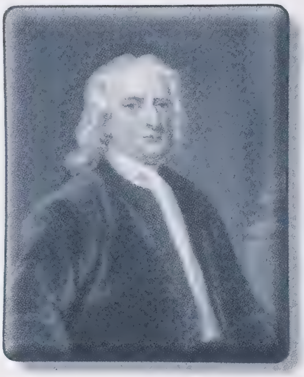
Date	Mathematician	$7x^2$	$5x^3$
1559	Jean Buteo	7◇	5❒
1585	Simon Stevin	7②	5③
1590	François Viète	7Q	5C
1610	Pietro Cataldi	$7\ \rlap{\,\mid}2$	$5\ \rlap{\,\mid}3$
1619	Jobst Bürgi	$\overset{ii}{7}$	$\overset{iii}{5}$

Finally, in 1637, a French mathematician by the name of René Descartes introduced the notation that exists today for powers with whole-number exponents.



CORBIS–BETTMANN

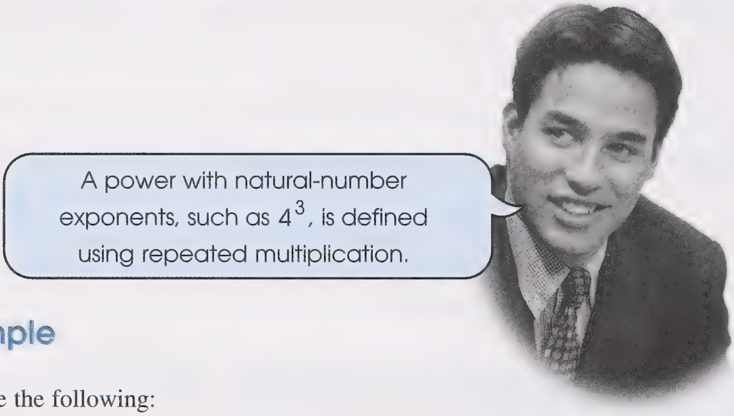
René Descartes



CORBIS–BETTMANN

Sir Isaac Newton

Later in the same century, an English mathematician by the name of Sir Isaac Newton extended Descartes's system of writing powers to include negative and fractional exponents.


A power with natural-number exponents, such as 4^3, is defined using repeated multiplication.

Example

Evaluate the following:

a. 4^3 **b.** $(-3)^2$ **c.** $\left(\frac{2}{3}\right)^4$

Solution

a. $$4^3 = 4 \times 4 \times 4 \\ = 64$$

b. $$(-3)^2 = (-3)(-3) \\ = 9$$

c. $$\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) \\ = \frac{16}{81}$$

A power with a zero exponent has a special meaning; it equals 1.

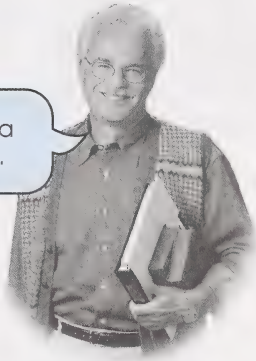
Example

Evaluate the following:

a. 5^0 **b.** $(-3)^0$ **c.** $\left(\frac{1}{4}\right)^0$ **d.** a^0

Solution

a. $5^0 = 1$ **b.** $(-3)^0 = 1$ **c.** $\left(\frac{1}{4}\right)^0 = 1$ **d.** $a^0 = 1$



A power with a negative exponent has a special meaning; it indicates a fraction.

Example

Evaluate the following powers:

a. 6^{-2} **b.** $(-2)^{-3}$ **c.** $\left(\frac{2}{3}\right)^{-1}$

Solution

a.
$$\begin{aligned}6^{-2} &= \frac{1}{6^2}\\ &= \frac{1}{36}\end{aligned}$$

b.
$$\begin{aligned}(-2)^{-3} &= \frac{1}{(-2)^3}\\ &= \frac{1}{-8}\\ &= -\frac{1}{8}\end{aligned}$$

c.
$$\begin{aligned}\left(\frac{2}{3}\right)^{-1} &= \frac{1}{\frac{2}{3}}\\ &= 1 \div \frac{2}{3}\\ &= 1 \times \frac{3}{2}\\ &= \frac{3}{2}\end{aligned}$$

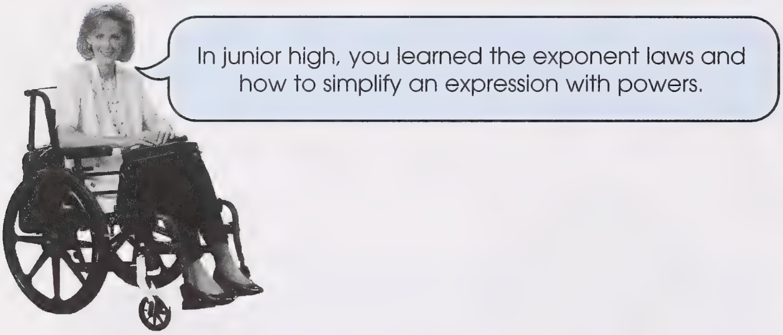
1. Turn to page 33 of *MATHPOWER™ 10* and answer the following questions:

   a. questions 13 to 16 of "Practice"
   b. questions 19 and 20 of "Practice"

2. Evaluate the following powers.

   a. $\left(\frac{1}{5}\right)^{-2}$ **b.** $\left(\frac{8}{9}\right)^{-1}$ **c.** $\left(-\frac{3}{7}\right)^{0}$ **d.** $\left(\frac{11}{12}\right)^{0}$

Compare your responses with those in the Appendix, Section 1: Activity 3.


In junior high, you learned the exponent laws and how to simplify an expression with powers.

To review the exponent laws, turn to page 30 of *MATHPOWER™ 10* and read from the red line to the red line on page 31.

3. Turn to page 33 of *MATHPOWER™ 10* and answer the following:

   a. questions 1 to 4, 9, and 11 of "Practice"

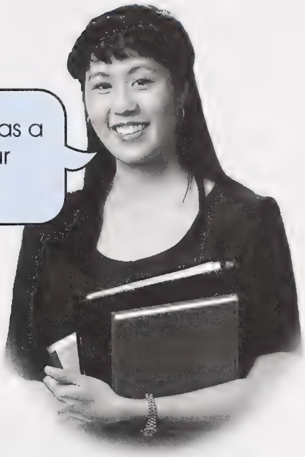
   b. question 84 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1, Activity 3.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

A power with a rational exponent also has a specialized meaning. You will use your calculator to discover the meaning.



4. **a.** Use the power key on your calculator to complete the following table. **Note:** On a graphing calculator, the power key is $\boxed{\wedge}$.

x	1	2	3	4	5	6	7	8	9
$x^{\frac{1}{2}}$									

b. Theorize on how $x^{\frac{1}{2}}$ is defined.

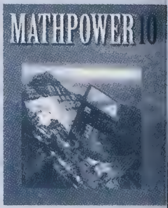
5. **a.** Use a graphing calculator to complete the following table:

x	1	2	3	4	5	6	7	8	9
$x^{\frac{1}{3}}$									

b. Theorize on how $x^{\frac{1}{3}}$ is defined.

Compare your responses with those in the Appendix, Section 1: Activity 3.

You have discovered that $x^{\frac{1}{2}} = \sqrt{x}$ and $x^{\frac{1}{3}} = \sqrt[3]{x}$. You will now use reasoning to prove that $x^{\frac{1}{n}} = \sqrt[n]{x}$, where n is a natural number.



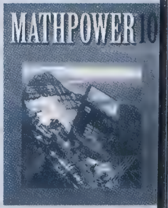
6. Turn to pages 34 and 35 of *MATHPOWER™ 10* and answer the following:

a. questions 1 to 4 of "Explore: Complete the Statements"
b. questions 1 to 3 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 3.

From your work with radicals, you can make the following conclusions about $x^{\frac{1}{n}}$.

- If n is an even number, $x \geq 0$. For example, if $n = 2$ and $x = -4$, then $(-4)^{\frac{1}{2}} = \sqrt{-4}$. There is no real square root of -4.
- If n is an odd number, x can be any real number. For example, if $n = 3$ and $x = -8$, then $(-8)^{\frac{1}{3}} = \sqrt[3]{-8}$. The cube root of -8 is -2.

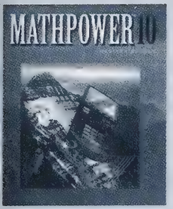


To discover how to evaluate expressions that have exponents in the form of $\frac{1}{n}$ or $-\frac{1}{n}$, turn to page 35 of *MATHPOWER™ 10* and work through Example 1.

7. Answer the following questions on page 37 of the textbook:

a. questions 1, 7, and 8 of "Practice"
b. questions 13 to 15 of "Practice"
c. questions 25 to 36 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 3.

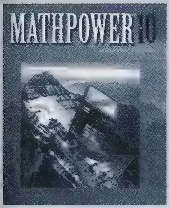


To discover the meaning of exponents in the form $\frac{m}{n}$ and how to evaluate powers such as $(-8)^{\frac{4}{3}}$ or $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$, turn to page 36 of *MATHPOWER™ 10* and read the explanation. Then work through Example 2.

8. Answer questions 2, 5, and 6 of "Practice" on page 37 of the textbook. **Note:** Use two methods.

9. Write each of the following expressions using exponents:

a. $\sqrt{5^3}$ **b.** $\left(\sqrt{7}\right)^5$ **c.** $\sqrt[3]{2^4}$ **d.** $\left(\sqrt[4]{3}\right)^5$



10. Turn to page 37 of *MATHPOWER™ 10* and answer questions 37, 39, 41, 43, 45, and 47 of "Practice."

Compare your responses with those in the Appendix, Section 1: Activity 3.

To evaluate expressions with rational exponents, use the power laws to simplify the expressions first.

Example

Evaluate each of the following:

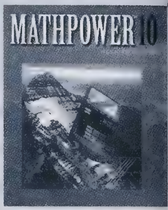
a. $8^{\frac{1}{2}} \times 8^{\frac{1}{6}}$ **b.** $3^{\frac{2}{3}} \div 9^{\frac{1}{3}}$

Solution

a.
$$\begin{aligned} 8^{\frac{1}{2}} \times 8^{\frac{1}{6}} &= 8^{\frac{1}{2}+\frac{1}{6}} \\ &= 8^{\frac{3}{6}+\frac{1}{6}} \\ &= 8^{\frac{4}{6}} \\ &= 8^{\frac{2}{3}} \\ &= \left(\sqrt[3]{8}\right)^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

b.
$$\begin{aligned} 3^{\frac{2}{3}} \div 9^{\frac{1}{3}} &= 3^{\frac{2}{3}} \div \left(3^2\right)^{\frac{1}{3}} \\ &= 3^{\frac{2}{3}} \div 3^{2\times\frac{1}{3}} \\ &= 3^{\frac{2}{3}} \div 3^{\frac{2}{3}} \\ &= 3^{\frac{2}{3}-\frac{2}{3}} \\ &= 3^0 \\ &= 1 \end{aligned}$$

11. Turn to page 37 of *MATHPOWER™ 10* and answer questions 52, 54, 56, and 64 of “Practice.”



Compare your responses with those in the Appendix, Section 1: Activity 3.

Evaluating an expression with nested radicals is similar to evaluating an expression with nested brackets.

Mathematical Process

☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

- When you evaluate a mathematical expression with nested brackets, work from the inside out.

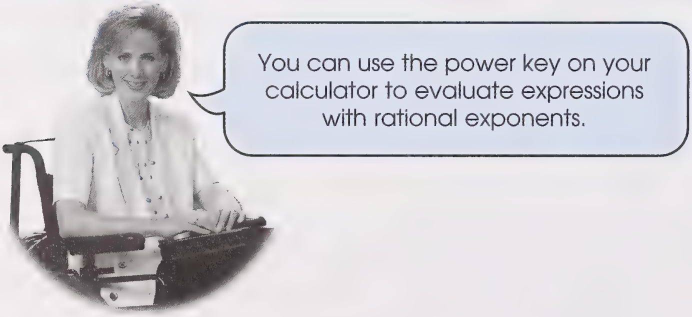
$$\begin{aligned} 3[5(7-1)] &= 3[5(6)] \\ &= 3\times 30 \\ &= 90 \end{aligned}$$

- When you evaluate an expression with nested radicals, work from the inside out.

$$\begin{aligned} \sqrt{\sqrt{81}} &= \sqrt{9} \\ &= 3 \end{aligned}$$

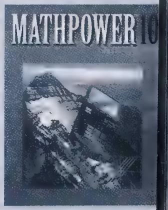
12. Turn to page 37 of *MATHPOWER™ 10* and answer questions 66 and 67 of “Practice.”

Compare your responses with those in the Appendix, Section 1: Activity 3.


You can use the power key on your calculator to evaluate expressions with rational exponents.

Mathematical Process

☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☒ Technology
☐ Visualization



Turn to page 37 of *MATHPOWER™ 10* and work through Example 3.

13. Answer questions 85, 87, and 89 of “Practice” on page 38 of the textbook.

14. Turn to “CONNECTING MATH AND PHYSIOLOGY” on pages 44 and 45 of *MATHPOWER™ 10* and answer the following:

a. questions 1 and 2 of Investigation 1, “Calculating Speeds”
b. questions 1 and 2 of Investigation 2, “Calculating Masses”

Compare your responses with those in the Appendix, Section 1: Activity 3.

Now Try This

Have you ever looked at distant planets and stars through a telescope? Have you ever watched an eclipse of the Sun or the Moon? Did you know that astronomers study mathematics as well as science?

To learn more about the connections between astronomy and mathematics, turn to “CAREER CONNECTION” on page 43 of *MATHPOWER™ 10* and read “Astronomy.”

JPL

15. Answer the following questions on page 43 of the textbook:

a. question 1 of Investigation 1, “Comparing Distances”
b. question 1 of Investigation 2, “Comparing Sizes”

Compare your responses with those in the Appendix, Section 1: Activity 3.

Looking Back

In this activity, you worked with rational exponents.

In your journal, explain the meaning of a zero exponent, a negative exponent, and a rational exponent.

Mathematical Process

☒ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

If you had difficulty writing an entire radical as a mixed radical, you may find it easier to factor the radicand completely, group the common factors, and simplify.

Example

Write each of the following as a mixed radical:

a. $\sqrt{48}$ **b.** $\sqrt[3]{48}$

Solution

a.

$$\begin{aligned}\sqrt{48} &= \sqrt{6\times 8}\\ &= \sqrt{2\times 3\times 2\times 2\times 2}\\ &= \sqrt{2\times 2\times 2\times 2\times 3}\\ &= \sqrt{2\times 2}\times\sqrt{2\times 2}\times\sqrt{3}\\ &= 2\times 2\times\sqrt{3}\\ &= 4\sqrt{3}\end{aligned}$$

b.

$$\begin{aligned}\sqrt[3]{48} &= \sqrt[3]{6\times 8}\\ &= \sqrt[3]{2\times 3\times 2\times 2\times 2}\\ &= \sqrt[3]{2\times 2\times 2\times 2\times 3}\\ &= \sqrt[3]{2\times 2\times 2}\times\sqrt[3]{2\times 3}\\ &= 2\sqrt[3]{6}\end{aligned}$$

1. **a.** Turn to page 46 of *MATHPOWER™ 10* and answer questions 21 to 23 of "Review."

b. Turn to page 48 of *MATHPOWER™ 10* and answer questions 6 to 8 of "Chapter Check."

2. Write each of the following entire radicals as a mixed radical:

a. $\sqrt[3]{54}$ **b.** $\sqrt[3]{16}$ **c.** $\sqrt[3]{24}$ **d.** $\sqrt[3]{81}$

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

If you had difficulty changing a radical expression to a power, you may find the following information helpful.

- The radical expression $\sqrt[3]{2^5}$ can also be written as $\left(\sqrt[3]{2}\right)^5$.
- The radicand will be the base of the equivalent power.
- The index of the radical sign will be the denominator in the rational exponent of the equivalent power. For example,

  $$\sqrt[3]{2} = 2^{\frac{1}{3}}$$

- The exponent in the radical expression will be the numerator in the rational exponent of the equivalent power. (If there is no exponent in the radical expression, the numerator will be 1.) For example,

  $$\sqrt[3]{2^5} = 2^{\frac{5}{3}}$$

Example

Write each of the following radicals as a power:

a. $\left(\sqrt[4]{5}\right)^3$ **b.** $\left(\sqrt[3]{2}\right)^5$ **c.** $\sqrt[3]{4^2}$ **d.** $\sqrt[4]{3^5}$

Solution

a. $\left(\sqrt[4]{5}\right)^3 = 5^{\frac{3}{4}}$ **b.** $\left(\sqrt[3]{2}\right)^5 = 2^{\frac{5}{3}}$ **c.** $\sqrt[3]{4^2} = 4^{\frac{2}{3}}$ **d.** $\sqrt[4]{3^5} = 3^{\frac{5}{4}}$

3. Write the following radicals as powers:

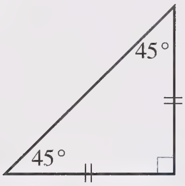
a. $\left(\sqrt[3]{27}\right)^2$ **b.** $\left(\sqrt{36}\right)^3$ **c.** $\left(\sqrt[4]{625}\right)^3$

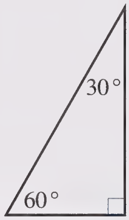
d. $\sqrt[3]{2^4}$ **e.** $\sqrt{5^3}$ **f.** $\sqrt[5]{(-5)^4}$

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

Enrichment

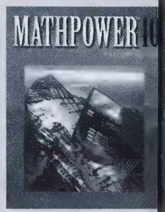
There are two right triangles that are called special right triangles: the 45°–45°–90° triangle and the 30°–60°–90° triangle.





The hypotenuses in each of these triangles is often expressed as a radical.

Turn to "INVESTIGATING MATH" on pages 26 and 27 of *MATHPOWER*™ *10* and do the following:



1. **a.** Answer questions 1 and 2 of Investigation 1, "The 45°–45°–90° Triangle."

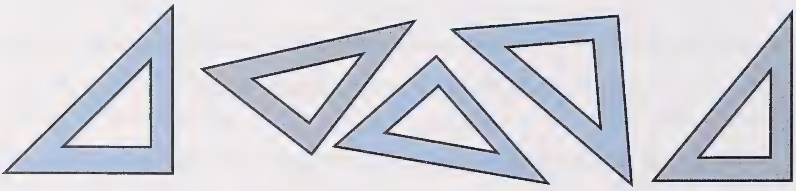
   b. Use the relationship between the length of the hypotenuse and the length of the leg to answer questions 3 to 5 of Investigation 1, "The 45°–45°–90° Triangle."

2. **a.** Answer question 2 of Investigation 2, "The 30°–60°–90° Triangle."

   b. Use the relationship between the length of the hypotenuse and the length of the side opposite the 30° angle to answer questions 1 and 3 of Investigation 2, "The 30°–60°–90° Triangle."

   c. Use the relationship between the length of the side opposite the 60° angle and the side opposite the 30° angle to answer questions 4 to 6 of Investigation 2, "The 30°–60°–90° Triangle."

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.



Conclusion

In this section, you investigated irrational numbers, which together with rational numbers form the real number system. You discovered that some irrational numbers can be written as a radical or as a power with a rational exponent. You then performed operations on irrational numbers.

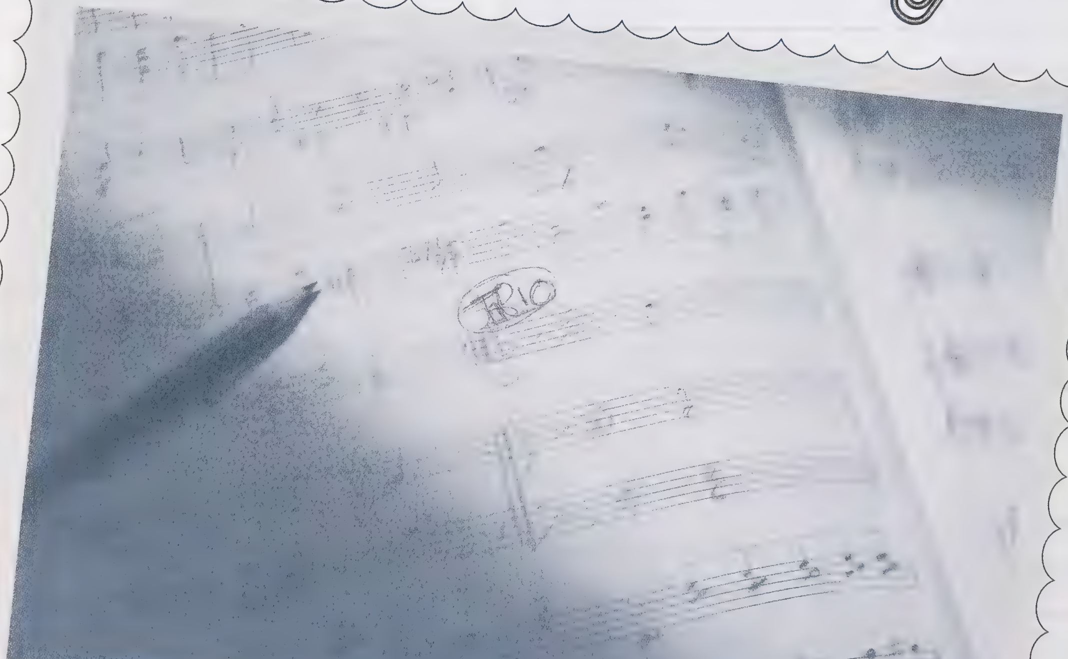
Like all systems—such as a communication system, Canada's postal system, and the nervous system in your body—the number system has interdependent parts that regularly interact to form a unified whole.

With what other systems are you familiar?

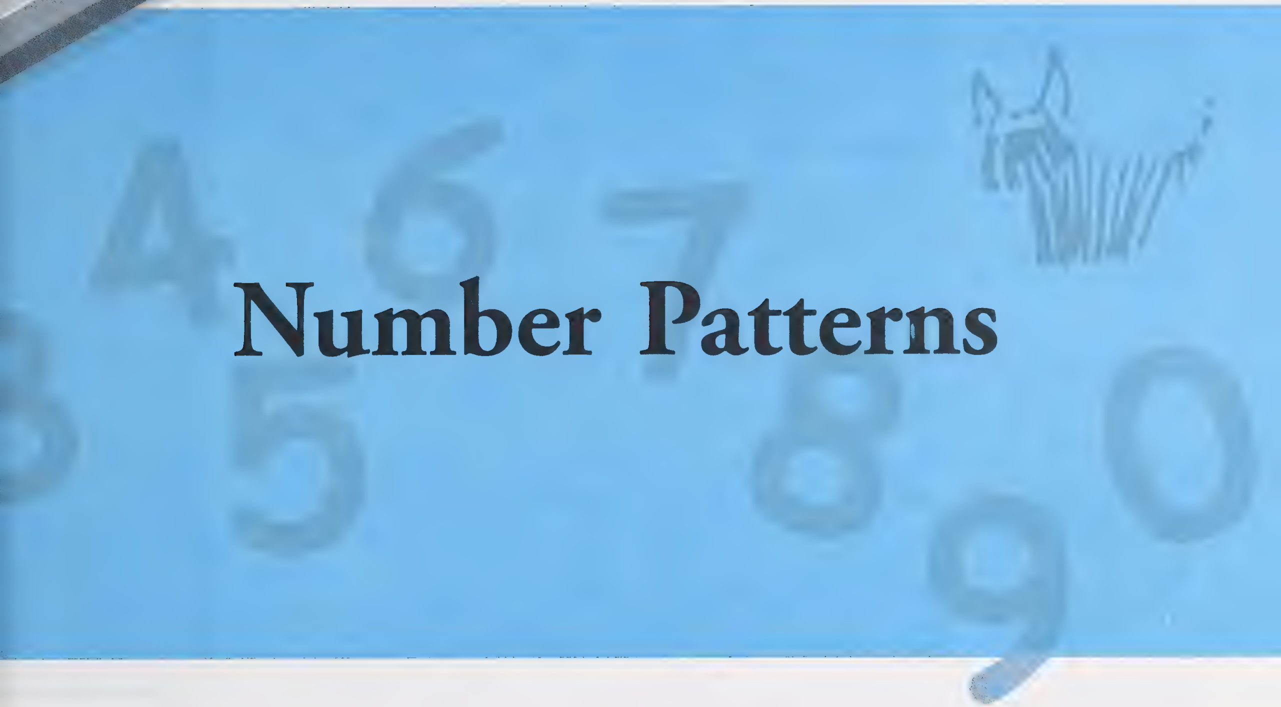
Assignment

Turn to Assignment Booklet 4A and complete the assignment for Section 1.

Section 2



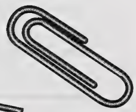
Number Patterns



Do you play the piano or some other musical instrument? Composers use a sequence of notes to create tunes. These melodies are then transcribed into sheet music. The pattern of notes in a piece of sheet music can be very complicated.

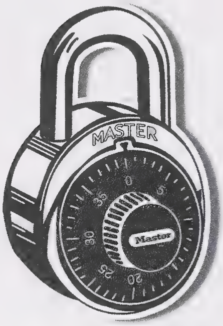
In number sequences—such as 5, 7, 9, 11 and 2, 6, 18, 54—the pattern is more easily described.

In this section, you will explore the pattern of number sequences. You will write formulas for generating sequences and draw graphs to describe the patterns. You will then calculate the sum of the terms of a sequence. In addition, you will review problem-solving and mental-math strategies and discover math connections.



Activity 1: Sequences

Have you ever used a combination lock on a locker or bicycle? If you have, you know that in order to open a combination lock you must perform the following sequence of actions:



Step 1: Start at zero (0).

Step 2: Turn to the right two whole turns and stop at your first combination number.

Step 3: Turn left one whole turn past your first number and stop at your middle number.

Step 4: Turn right and stop at your last number.

Step 5: Pull down on the lock and it should open.

There are many actions in everyday life that must be performed in a specified sequence. You know actions are sequential when words like "first," "then," and "next" are used.

The elements of a mathematical **sequence** are called **terms**. Each term is related to its **term number**.

sequence: an ordered list of elements (things in the set such as numbers, letters, or diagrams)

term: an element of a sequence

term number: the number that indicates the term's position in a sequence

infinite sequence: a sequence that has no last term or no first term

The variable t and a subscript can be used to indicate a specific term in a sequence. For example, in the following sequence the 2nd term, t_2, is 10 and the 4th term, t_4, is 18.

$$6, 10, 14, 18, 22, \ldots$$

A sequence that has no last term is called an **infinite sequence**. An ellipsis (three dots) is used to indicate that the sequence continues indefinitely. An infinite sequence does not need to follow a pattern. For example, the decimal equivalent to π is an infinite sequence of digits that has no pattern.

$$\pi = 3.141\,592\,653\ldots$$

However, all the sequences in this activity do have a pattern. You will begin this activity by exploring sequences of numbers, letters, or diagrams. You will use logic to find the pattern in each sequence and then determine the next few terms in the sequence.

There may be more than one way to describe the pattern in the sequences that you are given.

Example

Describe the pattern of the following sequence:

1, 4, 9, 16, 25, …

Solution

Response 1

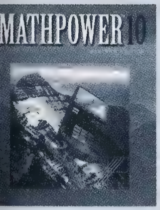
The sequence could have been generated by squaring the term numbers.

$$t_1 = 1^2 = 1$$
$$t_2 = 2^2 = 4$$
$$t_3 = 3^2 = 9$$
$$t_4 = 4^2 = 16$$
$$t_5 = 5^2 = 25$$

Response 2

The sequence could have been generated by starting with 1 and adding consecutive odd numbers.

$$t_1 = 1$$
$$t_2 = 1 + 3 = 4$$
$$t_3 = 1 + 3 + 5 = 9$$
$$t_4 = 1 + 3 + 5 + 7 = 16$$
$$t_5 = 1 + 3 + 5 + 7 + 9 = 25$$



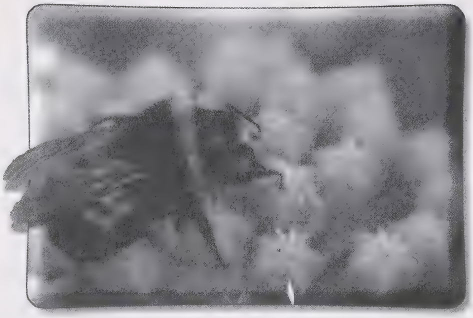
1. Turn to "GETTING STARTED" on page 52 of *MATHPOWER* ™ *10* and answer the following:

- **a.** questions 1 and 2 of Investigation 1, "Patterns in Numbers and Letters"
- **b.** questions 2 and 3 of Investigation 2, "Patterns in Diagrams"

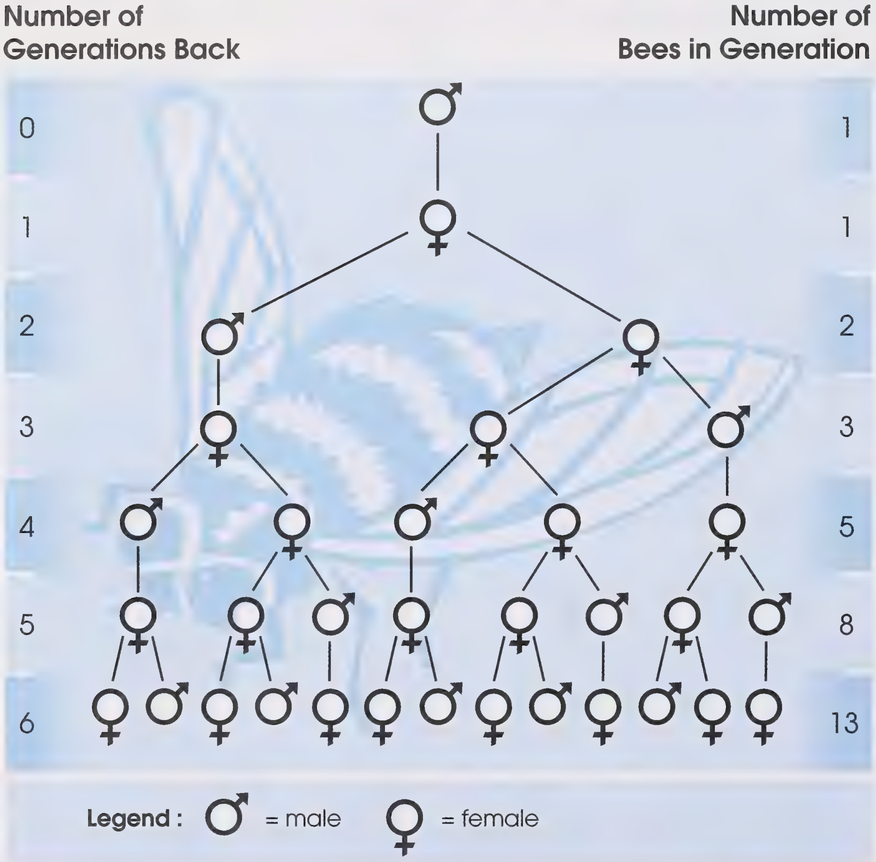
2. Turn to "INVESTIGATING MATH" on page 63 of *MATHPOWER* ™ *10* and answer questions 1 to 3 of Investigation 4, "Generating a Sequence."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Did you know that a male bee has only one parent—a mother—whereas a female bee has two parents—a mother and a father? As a result of this strange fact, the family tree of a male bee is very interesting.



The following chart shows the family tree of a male bee.


Number of Generations Back
Number of Bees in Generation
0
1
1
1
2
2
3
3
4
5
5
8
6
13
Legend : ♂ = male ♀ = female

The bee in the first generation back is the parent to the male bee; the bees in the second generation back are the grandparents to the male bee; the bees in the third generation back are the great grandparents to the male bee; and so on.

The number of bees in successive generations form this sequence:

1, 1, 2, 3, 5, 8, 13, …

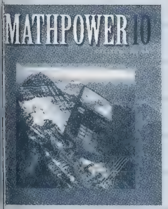
This sequence is named after Fibonacci, a great mathematician of the Middle Ages.

Notice that after the first two terms of the Fibonacci sequence, each successive term is the sum of the two preceding terms.

$$t_1 = 1$$
$$t_2 = 1$$
$$t_3 = t_1 + t_2 = 1 + 1 = 2$$
$$t_4 = t_2 + t_3 = 1 + 2 = 3$$
$$t_5 = t_3 + t_4 = 2 + 3 = 5$$
$$t_6 = t_4 + t_5 = 3 + 5 = 8$$
$$t_7 = t_5 + t_6 = 5 + 8 = 13$$

You may think that the Fibonacci sequence is a curiosity that is of interest only to mathematicians. In fact, the terms of the Fibonacci sequence, called the Fibonacci numbers, turn up in a surprising number of natural phenomena.

For example, the seeds in a pine cone, sunflower, and pineapple are arranged in clockwise and counterclockwise spirals. The number of seeds in each spiral are Fibonacci numbers.



To learn about other natural occurring spirals, turn to page 64 of *MATHPOWER™ 10* and read “Explore: Use the Data.”

3. Answer questions 1 to 7 of “Inquire” on page 65 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 1.

Generalizing the Rules for Sequences

Do you like number tricks? In a number trick, a sequence of directions is given. Usually, a person is told to choose a number and perform various operations to it. At the end of the trick, the result is the same, regardless of the number chosen.

Here is an example of a number trick:

• Choose a number.	4	7	12
• Triple the number.	12	21	36
• Add six.	18	27	42
• Divide by three.	6	9	14
• Subtract the number you chose.	2	2	2
• What is your answer? Did you get 2?			

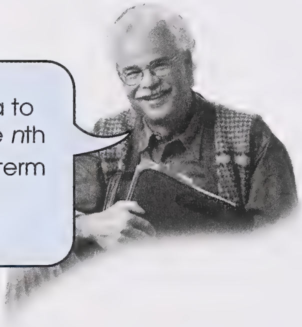
You can use algebra to generalize the directions for the number trick and prove that the trick always works. Let n represent the number chosen.

• Choose a number.	n
• Triple the number	$3n$
• Add six.	$3n+6$
• Divide by three.	$n+2$
• Subtract the number you chose.	2

This generalization proves that the result is always 2.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization



Sometimes, you can use algebra to generalize the rules for finding the nth term, t_n, of a sequence. The nth term is called the **general term** of the sequence.

general term: the nth term of a sequence; the formula that relates a term to its position in the sequence

Example

Write a formula to find the nth term of the following sequences:

a. 2, 5, 8, 11, …
b. 1, 3, 9, 27, …

Solution

Step 1: Make a table relating each term to its position in the sequence.

a.

Term Number (n)	1	2	3	4
Term (t_n)	2	5	8	11

b.

Term Number (n)	1	2	3	4
Term (t_n)	1	3	9	27

Step 2: Find a pattern in the growth of the terms.

a.

Term Number (n)	1	2	3	4
Term (t_n)	2	5	8	11

$+3 \quad +3 \quad +3$

b.

Term Number (n)	1	2	3	4
Term (t_n)	1	3	9	27

$\times 3 \quad \times 3 \quad \times 3$

Step 3: Use the pattern you discovered in Step 2 and the guess and check strategy to write a formula relating the term to its position in the sequence.

a.

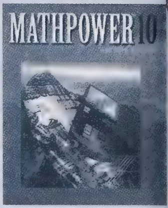
$$t_1 = 3(\mathbf{1}) - 1 = 3 - 1 = 2$$
$$t_2 = 3(\mathbf{2}) - 1 = 6 - 1 = 5$$
$$t_3 = 3(\mathbf{3}) - 1 = 9 - 1 = 8$$
$$t_4 = 3(\mathbf{4}) - 1 = 12 - 1 = 11$$

The formula is $t_n = 3n - 1$.

b.

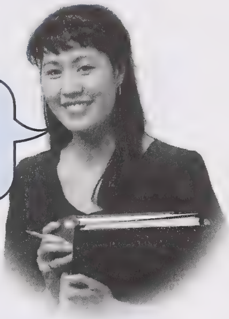
$$t_1 = 3^{\mathbf{1}-1} = 3^0 = 1$$
$$t_2 = 3^{\mathbf{2}-1} = 3^1 = 3$$
$$t_3 = 3^{\mathbf{3}-1} = 3^2 = 9$$
$$t_4 = 3^{\mathbf{4}-1} = 3^3 = 27$$

The formula is $t_n = 3^{n-1}$.



4. Turn to page 66 of *MATHPOWER™ 10* and answer questions 7, 9, 11, and 13 of "Practice." **Note:** Do not calculate the next three terms of each sequence.

Compare your responses with those in the Appendix, Section 2: Activity 1.



Given the general term of a sequence, you can find a specific term of a sequence without calculating the other terms.

Example

The general term of a sequence is $t_n = 3n - 1$. Generate the 5th term of the sequence.

Solution

$$\begin{aligned} t_n &= 3n - 1 \\ t_5 &= 3(5) - 1 \\ &= 15 - 1 \\ &= 14 \end{aligned}$$

The 5th term of the sequence is 14.

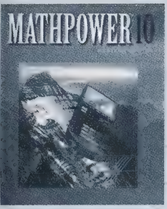
5. Turn to page 67 of *MATHPOWER™ 10* and answer questions 28, 30, 32, and 34 of "Practice."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

Given the general term of a sequence, you can generate the first few terms of the sequence and then graph the sequence. Graphs help you visualize the growth of the terms of a sequence.



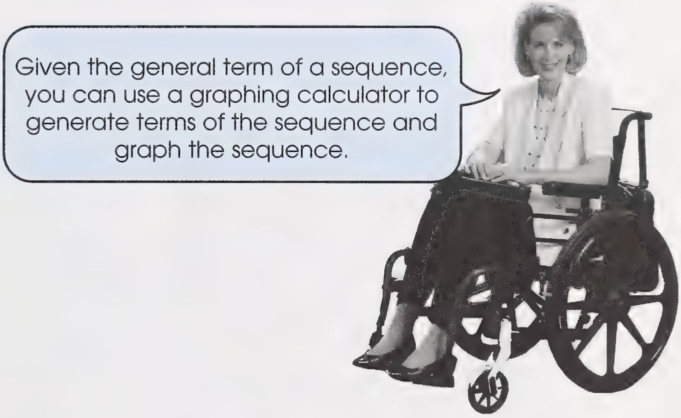
Turn to page 65 of *MATHPOWER™ 10* and work through Example 1 and Solution 1. **Note:** The graph of the sequence defined by the formula $t_n = 3n - 2$ shows that the terms are growing at a constant rate. The graph of the sequence defined by the formula $t_n = n^2 + 1$ shows that the terms are growing at an increasing rate.

6. Turn to page 66 of *MATHPOWER™ 10* and answer questions 1, 3, and 5 of "Practice."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☒ Visualization


Given the general term of a sequence, you can use a graphing calculator to generate terms of the sequence and graph the sequence.

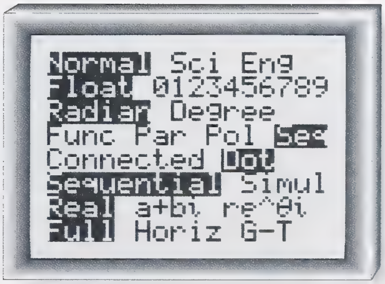
Example

The general term of a sequence is $t_n = 3n - 1$. Use a graphing calculator to generate the first five terms of the sequence and graph t_n versus n.

Solution

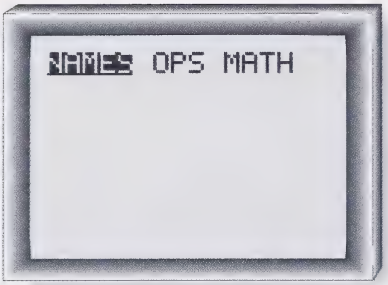
Step 1: Press MODE. Then use the arrow keys to select "Seq" (on the fourth line) and press ENTER. Also, select "Dot" (on the fifth line) and press ENTER. The screen will look like the one on the right.

The calculator is now in the sequence graphing mode.

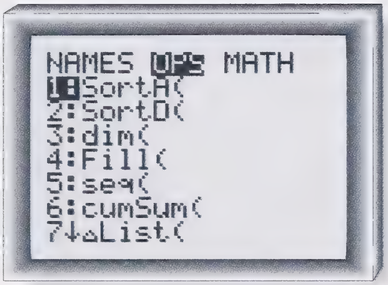


```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

Step 2: To list the first five terms of the given sequence, press 2nd [LIST]. The screen will look like this:

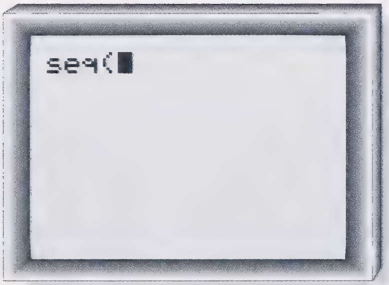


Use the arrow keys to select the OPS menu. This will bring up the following menu items:



The arrow at the bottom of the screen indicates that there are additional menu items. To view these items, simply scroll down using the arrow keys.

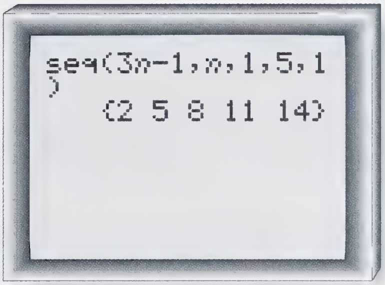
Press 5 to select "seq(." This action pastes "seq(" to the screen.



The blinking cursor indicates that the calculator is waiting for you to key in the following information: the general term, the variable, the first term number, the ending term number, and the incremental change in the variable (where the term numbers increase by increments of 1). **Note:** You must separate the information in the list with commas. After you have keyed in the information add the closing parenthesis.

Press the following sequences of keys:

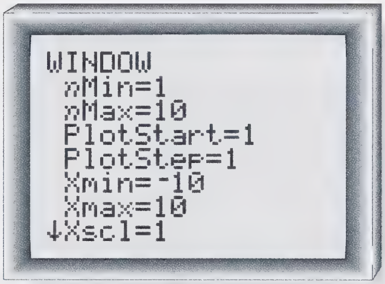
3 X,T,Θ,*n* − 1 , X,T,Θ,*n* , 1 , 5 , 1)
ENTER



```
seq(3n-1,n,1,5,1
)
    {2 5 8 11 14}
```

The first five terms are 2, 5, 8, 11, and 14.

Step 3: Display the default settings for sequence graphing by pressing WINDOW.



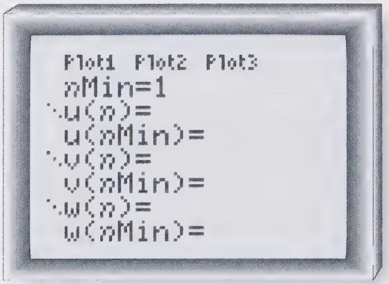
```
WINDOW
 nMin=1
 nMax=10
 PlotStart=1
 PlotStep=1
 Xmin=-10
 Xmax=10
↓Xscl=1
```

The arrow at the bottom of the screen indicates that you must use the arrow keys to scroll down and read the rest of the settings.

Use the arrow keys, the delete key, DEL, and the insert key, 2nd [INS], to make the following edits:

- The term numbers are 1, 2, 3, 4, and 5. Therefore, use the settings $n\text{Min} = 1$ and $n\text{Max} = 5$.
- The term numbers will be plotted on the x-axis. Therefore, use the settings $\text{Xmin} = 0$ and $\text{Xmax} = 5$.
- The terms will be plotted on the y-axis. The first five terms are 2, 5, 8, 11, and 14. Therefore, use the settings $\text{Ymin} = 0$ and $\text{Ymax} = 14$.

Step 4: Display the sequence Y = editor by pressing Y=.

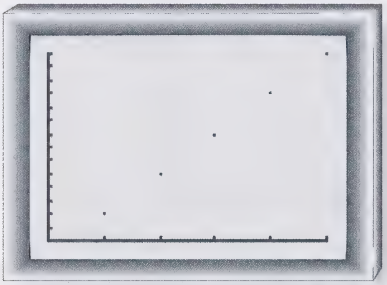


```
Plot1 Plot2 Plot3
nMin=1
·.u(n)=
 u(nMin)=
·.v(n)=
 v(nMin)=
·.w(n)=
 w(nMin)=
```

The TI-83 has three sequence functions: u(n), v(n), and w(n). You will only use u(n). Ensure that the icon to the left of "$\mathrm{u}(n)=$" is three dots. Use the arrow keys to move the cursor to the right of "$\mathrm{u}(n)=$". Then press the following key sequence:

3 X,T,Θ,n − 1

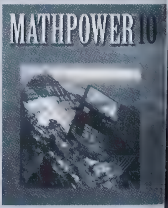
Step 5: Graph t_n versus n by pressing GRAPH. The graph will look like this:



Note: The value of each term, t_n, is related to its term number, n. This relation is a linear function. Because the domain of the linear function is the set of natural numbers, the points on the graph are not joined.

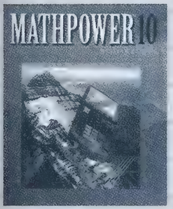
7. Turn to page 66 of *MATHPOWER™ 10* and answer questions 1, 3, and 5 of "Practice." **Note:** Use a graphing calculator.

Compare your responses with those in the Appendix, Section 2: Activity 1.



recursion formula: a formula in which a term of a sequence may be determined from the previous term

seed term(s): the first term(s) used to generate successive terms



It is sometimes convenient to describe the pattern of a sequence in terms of getting from one term to another, instead of starting with the general term. The pattern can be written as a **recursion formula**.

A recursion formula has two parts: a formula for the **seed term(s)** and a formula relating the general term to its preceding term(s).

Example

Write a formula to describe the following sequence; then use the formula to find the next three terms of the sequence.

3, 4, 7, 11, 18, 29, …

Note: This is the sequence in question 1.f. of Investigation 1, "Patterns in Numbers and Letters," on page 52 of *MATHPOWER™ 10.*

Solution

Step 1: Find a pattern in the growth of the terms.

$$\begin{array}{cccccc} 3, & 4, & 7, & 11, & 18, & 29 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ t_1 & t_2 & t_3=t_1+t_2 & t_4=t_2+t_3 & t_5=t_3+t_4 & t_6=t_4+t_5 \end{array}$$

Step 2: Use the pattern you discovered in Step 1 to write a recursion formula.

$t_1=3;\ t_2=4$ ⟵ These are the formulas for the seed terms.

$t_n=t_{n-2}+t_{n-1}$ ⟵ This is the formula that shows that the general term is the sum of the two preceding terms.

Step 3: Use the recursion formula to find the next three terms.

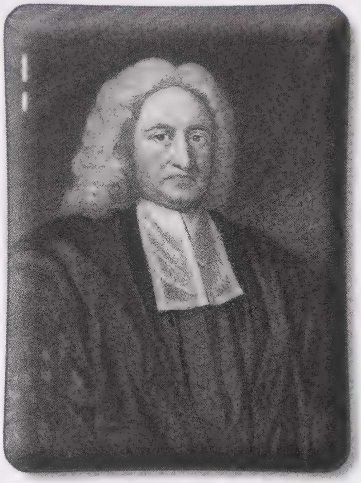
$$\begin{aligned} t_n &= t_{n-2}+t_{n-1} \\ t_7 &= t_{7-2}+t_{7-1} \\ &= t_5+t_6 \\ &= 18+29 \\ &= 47 \end{aligned} \qquad \begin{aligned} t_n &= t_{n-2}+t_{n-1} \\ t_8 &= t_{8-2}+t_{8-1} \\ &= t_6+t_7 \\ &= 29+47 \\ &= 76 \end{aligned} \qquad \begin{aligned} t_n &= t_{n-2}+t_{n-1} \\ t_9 &= t_{9-2}+t_{9-1} \\ &= t_7+t_8 \\ &= 47+76 \\ &= 123 \end{aligned}$$

The next three terms of the sequence are 47, 76, and 123.

8. Turn to page 67 of *MATHPOWER™ 10* and answer questions 35 to 37 of "Practice."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Real-World Applications of Sequences


CORBIS–BETTMANN

An English astronomer by the name of Edmund Halley (1656–1743) found that a comet he had seen in 1682 had an identical path to the paths taken by certain comets in 1456, 1531, and 1607. He hypothesized that the same comet was being observed, and he predicted the comet would reappear in 1758. When it did, it was named Halley's comet after him.

Various records and art works show that Halley's comet had been seen at regular intervals of approximately 76 years since 240 B.C. Therefore, predicting Halley's comet is a real-world application of sequences.

If you wish to read more about Edmund Halley and Halley's comet and you have access to the Internet, refer to the following websites:

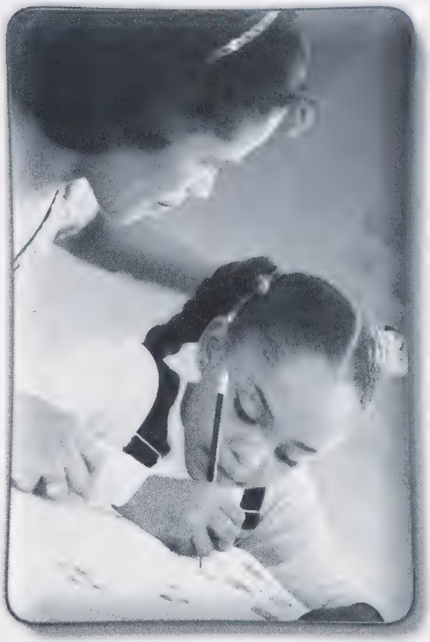
- **http://www.astro.cz/solar/eng/halley.htm**
- **http://sci.hirosaki-u.ac.jp/~hiro/tnp/nineplanets/halley.html**
- **http://www.phy.mtu.edu/apod/ap960706.html**

You may also use one of the Internet's search engines to locate additional information. Use the key word "Halley" for your search.

9. To explore other real-world applications of sequences, turn to page 67 of *MATHPOWER™ 10* and answer questions 46 and 47 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Arithmetic Sequences



Children first become aware of numbers through counting. Often, they recite counting rhymes like this:

One, two,
Buckle my shoe.
Three, four,
Close the door.

One potato, two potato,
Three potato four.
Five potato, six potato,
Seven potato more.

arithmetic sequence: a sequence in which each successive term, after the first term, can be found by adding the same number

The counting numbers form an **arithmetic sequence**.

1, 2, 3, 4, 5, 6, 7, 8, …

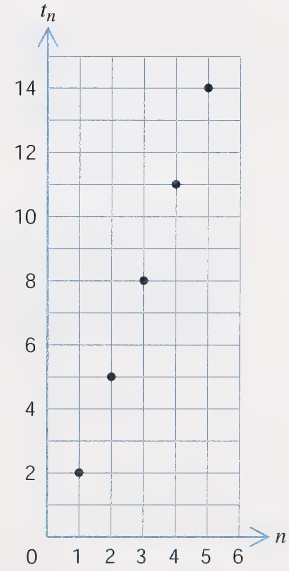
+1 +1 +1 +1 +1 +1 +1

An arithmetic sequence is a sequence in which each successive term may be found by adding the same number.

If the number added to each successive term of an arithmetic sequence is positive, the sequence grows at a constant rate.

For example, the arithmetic sequence 2, 5, 8, 11, 14, … is formed by adding +3 to each successive term. The graph at the right may help you visualize that this sequence is growing at a constant rate.

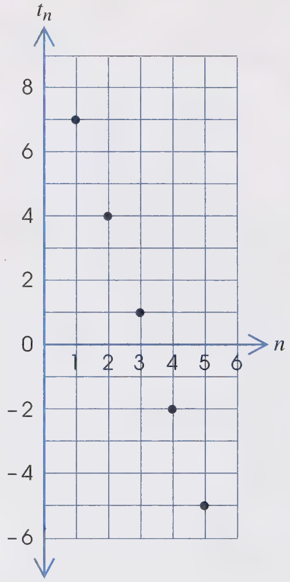
Note: The value of each term is related to its position in the sequence. This relation is a linear function. Because the domain of the linear function is the set of natural numbers, the points in the graph are not joined.



Mathematical Process
■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
☒ Visualization

If the number added to each successive term of an arithmetic sequence is negative, the sequence shrinks at a constant rate.

For example, the arithmetic sequence $7, 4, 1, -2, -5, \ldots$ is formed by adding -2 to each successive term. The graph at the right may help you visualize that this sequence is shrinking at a constant rate.

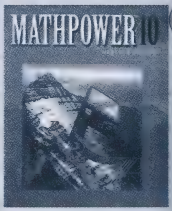


Note: The value of each term is related to its position in the sequence. This relation is a linear function. Because the domain of the linear function is the set of natural numbers, the points on the graph are not joined.

The first term of an arithmetic sequence is represented by a. The number added to each successive term of an arithmetic sequence is the difference between consecutive terms. It is called the **common difference** of the sequence and is represented by d.

common difference: the difference between two consecutive terms of an arithmetic sequence

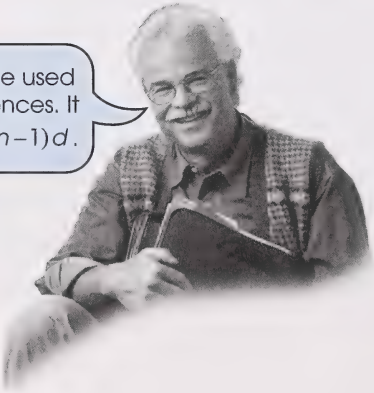
10. Turn to page 72 of *MATHPOWER™ 10* and answer the following questions:
    - **a.** "Explore: Complete the Table"
    - **b.** questions 1 to 4 of "Inquire"

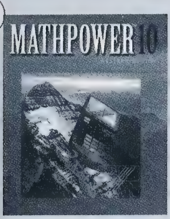


Compare your responses with those in the Appendix, Section 2: Activity 1.

Earlier in this activity you used a formula for the general term of a sequence to find the first few terms of a sequence. A general formula can be used when you are working with arithmetic sequences.

There is another method that can be used when working with arithmetic sequences. It involves using the formula $t_n = a + (n-1)d$.



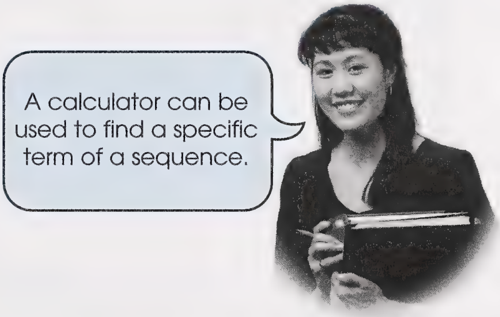


Turn to page 72 of *MATHPOWER™ 10* and read from the red line to the bottom of the page. Then work through Example 1 on page 73. Pay particular attention to Method 1.

11. Answer the following questions on pages 74 and 75 of the textbook:

- **a.** questions 1, 3, and 5 of "Practice"
- **b.** questions 7 to 11 of "Practice"
- **c.** questions 15 to 18 of "Practice"
- **d.** questions 23 to 27 of "Practice"

Compare your responses with those in the Appendix, Section 2: Activity 1.


A calculator can be used to find a specific term of a sequence.

Example

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Find the 25th term of the following sequence:

3, 7, 11, 15, 19, …

Solution

Step 1: Find the general term.

$a = 3$ and $d = 4$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 3 + (n-1)4 \end{aligned}$$

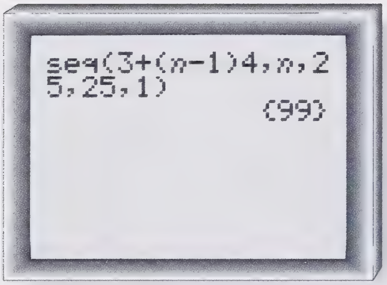
It is not necessary to simplify the general term.

Step 2: Press MODE; use the arrow keys to select "Seq" (on the fourth line); and press ENTER.

Step 3: Press 2nd [LIST], use the arrow keys to select the OPS menu, and press 5 to select “seq(.”

Press the following key sequence. **Note:** The first term number and the last term number are both 25. The incremental change is always 1.

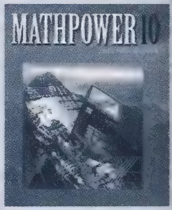
3 + (X,T,Θ,*n* − 1) 4 , X,T,Θ,*n* ,
2 5 , 2 5 , 1) ENTER



```
seq(3+(n-1)4,n,2
5,25,1)
                {99}
```

The 25th term is 99.

12. Turn to page 75 of *MATHPOWER*™ *10* and answer questions 28 to 30 of “Practice.” **Note:** Use a graphing calculator to find the terms.



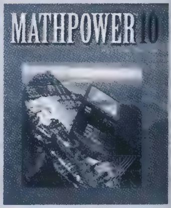
Compare your responses with those in the Appendix, Section 2: Activity 1.

Earlier in this activity, you dealt with infinite sequences. Sequences with a specific number of terms are called **finite sequences**.

finite sequence: a sequence that has both a first and last term

You can use the formula $t_n = a + (n-1)d$ to determine the number of terms in a finite arithmetic sequence.

Turn to page 73 of *MATHPOWER*™ *10* and work through Example 2.



13. Answer the following questions on pages 75 and 76 of the textbook:

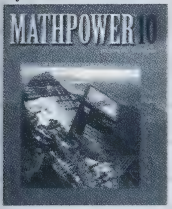
a. questions 32 to 35 of “Practice”
b. question 49, 50, 56, and 64 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 2: Activity 1.

The terms between two non-consecutive terms of an arithmetic sequence are called the **arithmetic means**.

arithmetic mean(s): the term(s) between two non-consecutive terms of an arithmetic sequence

You can use the formula $t_n = a + (n-1)d$ to determine the terms between the first term and last term of a finite arithmetic sequence.



Turn to page 73 of *MATHPOWER™ 10* and read from the red line that follows Example 2 to the red line on page 74, working through Example 3.

14. Answer questions 38 to 42 of "Practice" on page 75 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 1.

Geometric Sequences

Have you ever received a chain letter? Chain letters are fairly common, even though pyramid schemes are illegal.

Some chain letters work like this. You receive a letter with instructions and a list of names. You must send a specified amount of money or a specific type of good to the person at the top of the list, cross this person's name off the list, and add your name to the bottom of the list. You then send copies of the revised letter to the specified number of people.

A chain letter is a pyramid scheme because people taking part in the early stages of the scheme may make more money than they invest. Eventually, the scheme peters out and the people in the later stages of the scheme lose money.

THIS IS AN INNOVATION IN LETTER WRITING: A "PYRAMID CLUB." PLEASE MAKE 3 DUPLICATES OF THIS LETTER AND MAIL THEM TO ME. –
FORWARD THE ORIGINAL TO A FRIEND AND ASK HIM TO DO LIKEWISE. A COMPLETE EXPLANATION WILL BE FORTHCOMING, PENDING A SUCCESSFUL OUTCOME.
THANK YOU, WILEY.
AFTER A WHILE
HOW DID YOU EVER MAKE OUT WITH YOUR 'PYRAMID CLUB', WILEY?
GREAT.

[1]

From the use in which the letters in the cartoon were put, it looks as if Wiley's pyramid club was appropriately named.

---

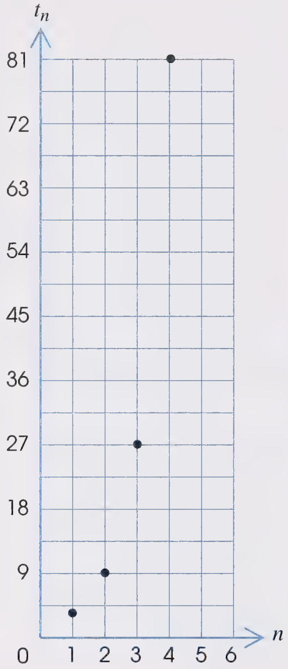
[1] Johnny Hart, cartoon (Los Angeles: Creators Syndicate, Inc. n.d.). Reprinted by permission of Johnny Hart and Creators Syndicate, Inc.

The sequence 1, 3, 9, 27, … is called a **geometric sequence**. Each successive term of a geometric sequence can be found by multiplying the preceding term by the same number.

Because each successive term of a geometric sequence is multiplied by the same number, the ratio of consecutive terms is a constant. The ratio of consecutive terms in a geometric sequence is called the **common ratio**.

If the common ratio of a geometric sequence is greater than 1, the sequence grows at an increasing rate.

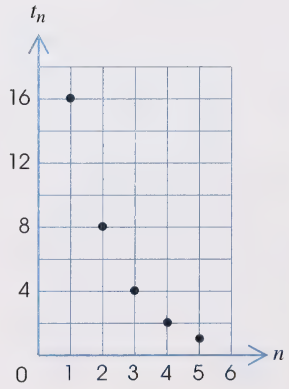
For example, the geometric sequence 1, 3, 9, 27, … is formed by multiplying by 3. The graph on the right may help you visualize that the sequence is growing at an increasing rate.



Remember: *Because the domain of the function is the set of natural numbers, the points on the graph are not joined.*

If the common ratio of a geometric sequence is less than 1, the sequence shrinks at a decreasing rate.

For example, the geometric sequence 16, 8, 4, 2, … is formed by multiplying by $\frac{1}{2}$. The graph on the right may help you visualize that the sequence is shrinking at a decreasing rate.



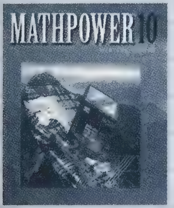
geometric sequence: a sequence in which each successive term, after the first term, can be found by multiplying by the same number

common ratio: the ratio of two consecutive terms in a geometric sequence

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

geometric mean(s): the term(s) between two non-consecutive terms of a geometric sequence



The terms between two non-consecutive terms of a geometric sequence are called the **geometric means**.

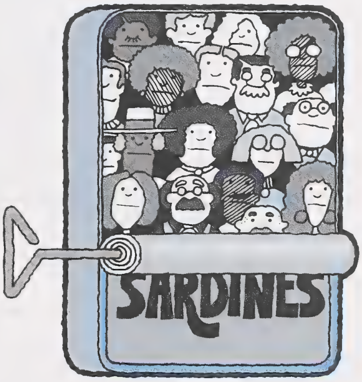
15. Turn to "INVESTIGATING MATH" on pages 84 and 85 of *MATHPOWER™ 10* and answer the following:

- **a.** questions 1, 3, 5, 7, 9, 11, and 13 of Investigation 1, "Writing Terms"
- **b.** questions 1, 3, 5, 7, and 9 of Investigation 2, "Finding Geometric Means"
- **c.** questions 1 to 3 of Investigation 3, "Applications"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Comparing Arithmetic and Geometric Sequences

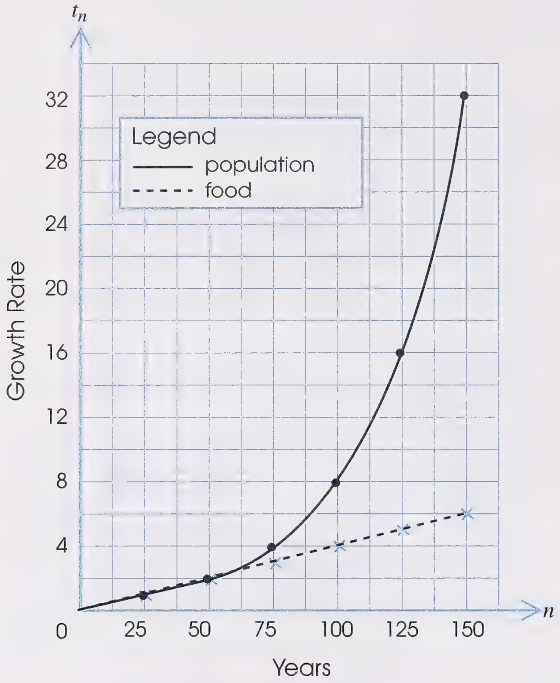
For many years, economists and scientists have been wondering about the effects of population growth.



In 1798, an English economist, Thomas Malthus, theorized that the human population, if unchecked, would double every 25 years. He used the geometric sequence 1, 2, 4, 8, 16, 32, 64, 128, 256, … to model the growth rate of the population over the next two centuries.

Malthus assumed that the amount of available food would increase at a constant rate over the same time period. He used the arithmetic sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, … to model the growth rate of food over the next two centuries.

The following graph uses Malthus's sequences to compare the growth rate of people and the amount of available food.



16. **a.** If there were one billion people on Earth in 1798, how many people did Malthus theorize could be on Earth in 1823? 1848? 1873? 1898? 1923? 1948? 1973? 1998?

b. Use the Internet or an almanac to research Earth's population in 1998.

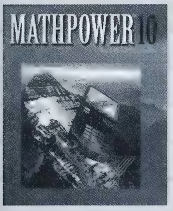
c. Was Malthus's theory correct?

17. Turn to page 85 of *MATHPOWER™ 10* and answer question 3 of Investigation 4, "Critical Thinking."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Now Try This

Mathematical Process
- ■ Communication
- ■ Connection
- ■ Estimation
- ☒ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization



It's time to further hone your mental math skills.

By now, you probably find that it is easy to mentally multiply and divide by powers of ten. You can use these skills to mentally multiply and divide by 5000, 500, 50, 5, 0.5, 0.05, or 0.005.

18. a. Turn to page 3 of *MATHPOWER*™ *10* and answer questions 1 to 23 of "Mental Math: Multiplying by Multiples of 5."

b. Turn to page 53 of *MATHPOWER*™ *10* and answer questions 1 to 26 of "Mental Math: Dividing by Multiples of 5."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process
- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ☒ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

Are you sometimes confused about how to solve a problem because it seems too complicated?

It is sometimes helpful to break the original problem into simpler problems, solve the simpler problems, look for a pattern, and then use the pattern to solve the original problem.

Example

Find the units digit in the standard form of 7^{50} without evaluating the power.

Solution

Step 1: Consider simpler versions of this problem and look for a pattern. **Note:** You do not need to evaluate each power; you only need to calculate the last digit.

$7^{1} =$ **7**
$7^{2} =$ **49**
$7^{3} =$ 343
$7^{4} =$ 2401

$7^{5} =$ ——— **7**
$7^{6} =$ ——— **9**
$7^{7} =$ ——— **3**
$7^{8} =$ ——— **1**

$7^{9} =$ ——— **7**
$7^{10} =$ ——— **9**
$7^{11} =$ ——— **3**
$7^{12} =$ ——— **1**

Notice that the units digit of the standard form of the powers of 7 form a cyclic pattern of four digits: 7, 9, 3, and 1.

Also, notice the following pattern:

- When the exponent is a multiple of 4, the units digit is 1.
- When the exponent is 1 more than a multiple of 4, the units digit is 7.
- When the exponent is 2 more than a multiple of 4, the units digit is 9.
- When the exponent is 3 more than a multiple of 4, the units digit is 3.

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

$7^5 = 7^{4+1} = \underline{\hspace{1cm}}7$

$7^6 = 7^{4+2} = \underline{\hspace{1cm}}9$

$7^7 = 7^{4+3} = \underline{\hspace{1cm}}3$

$7^8 = 7^{4+4} = \underline{\hspace{1cm}}1$

$7^9 = 7^{8+1} = \underline{\hspace{1cm}}7$

$7^{10} = 7^{8+2} = \underline{\hspace{1cm}}9$

$7^{11} = 7^{8+3} = \underline{\hspace{1cm}}3$

$7^{12} = 7^{8+4} = \underline{\hspace{1cm}}1$

Step 2: Extend the pattern to solve the original pattern.

$$7^{50} = 7^{48+2}$$

The exponent is 2 more than a multiple of 4.

Therefore, the units digit of 7^{50} is 9.

19. Without using a calculator, find the units digit in the standard form of 4^{75}.

Compare your response with the one in the Appendix, Section 2: Activity 1.

Looking Back

In this activity, you explored number patterns called sequences. You generated the terms of sequences using general terms and recursion formulas. You also graphed these sequences in the coordinate plane from tables of values and on your graphing calculator. In particular, you studied two special types of sequences: the arithmetic sequence and the geometric sequence.

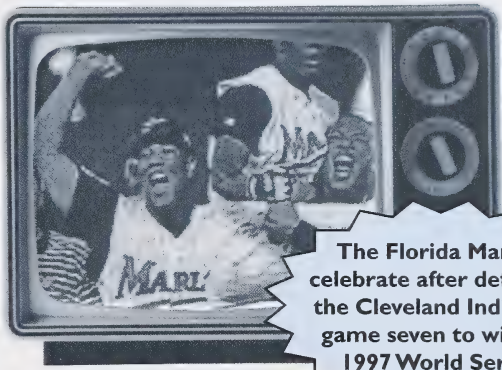
In your journal, describe how arithmetic and geometric sequences are similar and how they are different. Use examples to illustrate your explanation. Now, use your research skills to find out why the words "arithmetic" and "geometric" are used to name these sequences. Summarize your findings.

Mathematical Process

☒ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

Activity 2: Series

Do you follow professional baseball? Are you familiar with the World Series? Every professional and aspiring young baseball player dreams of winning the World Series—an event that decides who will be baseball's champions for a year. The World Series is a series of at most seven games between the two remaining teams, where the first team that wins four games becomes the World Series champions.



The Florida Marlins celebrate after defeating the Cleveland Indians in game seven to win the 1997 World Series.

series: the sum of the terms of a sequence

In baseball, a series is a sequence of games viewed as a whole. In mathematics, a **series** is the sum of the terms of a sequence. In this activity, you will examine series in mathematics.

1. The sum of consecutive odd numbers starting with 1 are squares.

$$1 = 1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4^2$$

$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

Use this pattern to find the sum of the following series:

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$$

2. a. Copy and find the sums of the following set of series.

$$1 + 1 =$$

$$1 + 1 + 2 =$$

$$1 + 1 + 2 + 3 =$$

$$1 + 1 + 2 + 3 + 5 =$$

$$1 + 1 + 2 + 3 + 5 + 8 =$$

$$1 + 1 + 2 + 3 + 5 + 8 + 13 =$$

$$1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 =$$

b. Compare the sums in question 2.a. with the following Fibonacci sequence:

$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$

What pattern do you notice?

c. Use the pattern that you discovered in question 2.b. to find the sum of the first 10 terms of the Fibonnacci sequence.

3. Read the following story about Carl Friedrich Gauss (1777–1855), one of the greatest mathematicians of all time.

> In school Carl was always causing trouble because the work was too easy for him and he was bored. Once his teacher decided to keep him quiet for a long time by asking him to add all the whole numbers from 1 to 99. The teacher's plan was spoiled because young Carl gave the answer 4950 immediately and without writing anything down!
>
> Here is the way Carl got the answer so quickly. He thought of the numbers 1 to 99 written forward and backward like this:
>
> $$\begin{array}{l} 1+\ 2+\ 3+\ 4+\cdots+96+97+98+99 \\ 99+98+97+96+\cdots+\ 4+\ 3+\ 2+\ 1 \end{array}$$
>
> Then he could see that each pair (1 and 99, 2 and 98, and so on) added up to 100. There were 99 such pairs and so the rows together added up to 99 times 100, or 9900. Because Carl wanted the sum of only one row, he divided 9900 by 2 in his head and got the answer 4950.[1]

Use Carl Friedrich Gauss's method to find the sum of the following series:

$1+2+3+4+\cdots+196+198+199$

Compare your responses with those in the Appendix, Section 2: Activity 2.

---

[1] Merle Mitchell, *Mathematical History: Activities, Puzzles, Stories, and Games*, 45. Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*, ©1978 by the National Council of Teachers of Mathematics.

Recipes are useful when baking. They provide the sequence of steps that a cook must follow.

Similarly, formulas are helpful to mathematicians.

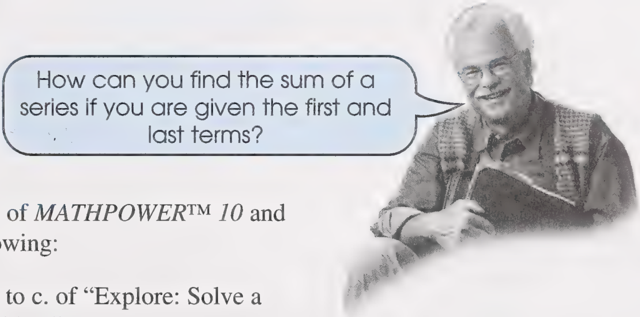
To discover a formula for the sum of an arithmetic series, turn to page 80 of *MATHPOWER™ 10.* Read from the red line to Example 1 on page 81 and work through Example 1.

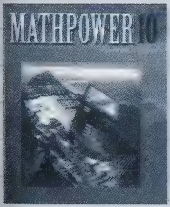
4. Answer the following:

   a. questions 5, 7, and 9 of "Practice" on page 82 of the textbook

   b. questions 23 and 27 of "Applications and Problem Solving" on page 83 of the textbook

Compare your responses with those in the Appendix, Section 2: Activity 2.


How can you find the sum of a series if you are given the first and last terms?



5. Turn to page 80 of *MATHPOWER™ 10* and answer the following:

   a. questions a. to c. of "Explore: Solve a Simpler Problem"
   b. questions 1 to 5 of "Inquire"

6. Turn to page 82 of *MATHPOWER™ 10* and work through Example 2. Then use formulas (1) and (2) to answer the following questions on page 83:

   a. questions 10, 12, and 14 of "Practice"
   b. question 31 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 2.

Mathematical Process

☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☒ Technology
☐ Visualization

You can use a graphing calculator to find the sum of a finite arithmetic series.

Example

Find the sum of the first 50 terms of the following sequence:

3, 8, 13, 18, 23, …

Solution

Step 1: Find the general term.

$$a = 3 \text{ and } d = 5$$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 3 + (n-1)5 \end{aligned}$$

Note: It is not necessary to simplify the general term.

Step 2: Press `MODE`; use the arrow keys to select "Seq" on the fourth line; and press `ENTER`.

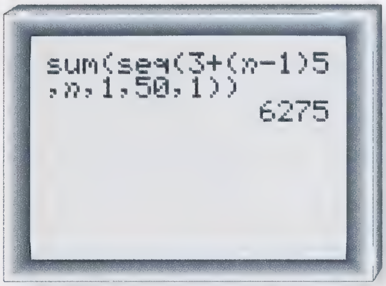
Step 3: Press `2nd` [LIST], use the arrow keys to select the MATH menu, and press `5` to select "sum(." This action pastes "sum(" onto the screen.

Step 4: Press `2nd` [LIST], use the arrow keys to select the OPS menu, and press `5` to select "seq(." This action pastes "seq(" onto the screen.

Then press the following key sequence:

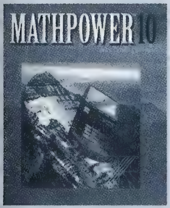
`3` `+` `(` `X,T,Θ,n` `−` `1` `)` `5` `,` `X,T,Θ,n` `,` `1` `,` `5` `0` `,` `1` `)`

Press) to close the parentheses after sum; then press ENTER.



```
sum(seq(3+(n-1)5
,n,1,50,1))
             6275
```

The sum is 6275.



7. Turn to page 82 of *MATHPOWER™ 10* and answer questions 6 and 8 of "Practice." **Note:** Use a graphing calculator.

Compare your responses with those in the Appendix, Section 2: Activity 2.

Now Try This

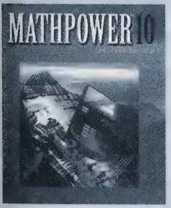
Have you ever read *Alice in Wonderland*? The author, Lewis Carroll, was also a mathematician who specialized in logic.

In this part of the activity, you will solve problems using logic.

Turn to "PROBLEM SOLVING" page 60 of *MATHPOWER™ 10* and read "Use Logic."

Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ☒ Problem Solving
- ☒ Reasoning
- ■ Technology
- ■ Visualization



8. Answer questions 1 and 5 of "Applications and Problem Solving." on page 61 of *MATHPOWER™ 10.*

I'm late.
I'm late.

Compare your responses with those in the Appendix, Section 2: Activity 2.

In mathematical logic, a statement is considered to be true or false—there is no middle ground. One example alone never proves that a statement is true; however, one counterexample proves that a statement is false.

Graphic organizers, such as Venn diagrams, may help you prove whether a statement is true or false.

Example

Is each of the following statements true or false? Explain your reasoning.

a. All decimal numbers are rational numbers.
b. All rectangles are parallelograms.

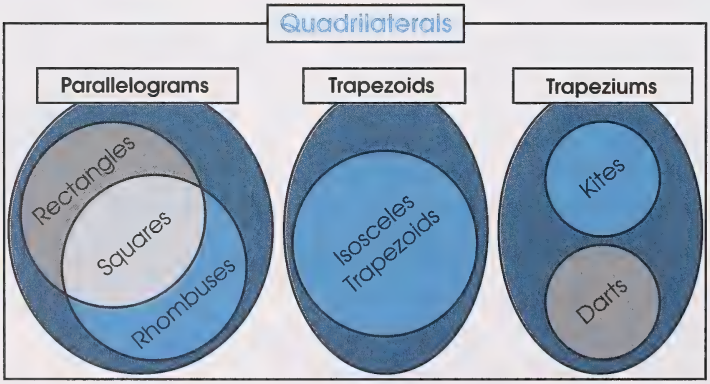
Solution

a. Decimal numbers include the following:

- terminating decimal numbers, such as 0.25
- repeating decimal numbers, such as 0.666…
- non-terminating and non-repeating decimal numbers, such as 0.010 110 111…

Since non-terminating and non-repeating decimal numbers are irrational numbers, the statement is false.

b. The following Venn diagrams show the relationship between the types of quadrilaterals.



Since the set of rectangles is inside the set of parallelograms, the statement is true.

9. State whether each of the following statements is **true** or **false**.

   a. All whole numbers are rational numbers.
   b. All integers are rational numbers.
   c. All rational numbers are integers.
   d. All radicals are irrational numbers.
   e. All squares are rhombuses.
   f. All rhombuses are squares.
   g. All trapezoids are quadrilaterals.
   h. All quadrilaterals are trapezoids.
   i. All sequences have a common difference.
   j. All sequences have a pattern.

Compare your responses with those in the Appendix, Section 2: Activity 2.

Now Try This

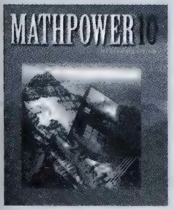
Mathematical Process

- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

There is an old saying that if you don't like the weather in Alberta, wait a minute. This saying is also applicable to many other regions in Canada.

Have you ever considered becoming a meteorologist? Meteorology students study mathematics and physics as well as atmospheric science.

To discover more about this profession, turn to "CAREER CONNECTION" on page 77 of *MATHPOWER*™ *10* and read "Meteorology."



10. Answer the following questions on page 77 of the textbook:

    a. question 2 of Investigation 1, "Precipitation in Canada"
    b. questions 1 to 3 of Investigation 2, "Greatest Precipitation"

Compare your responses with those in the Appendix, Section 2: Activity 2.

Looking Back

Mathematical Process

- ☒ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

In this activity, you explored series. You discovered that a series is the sum of the terms of a sequence.

The word "series" is often used in everyday conversation. You have heard people refer to a particular television series, a series of lectures, a series of articles, or even a series of accidents. In your journal, explain how these uses of the word "series" are similar to the mathematical meaning of the word and how these uses are different.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section, you worked with arithmetic sequences and geometric sequences.

An arithmetic sequence is a sequence in which the difference between consecutive terms is a constant called the common difference. The general arithmetic sequence is a, $a+d$, $a+2d$, $a+3d$, $\ldots, a+(n-1)d$, where a is the first term, d is the common difference, and n is the number of terms. Therefore, the formula for the nth term of an arithmetic sequence is $t_n = a+(n-1)d$.

A geometric sequence is a sequence in which the ratio of consecutive terms is a constant, called the common ratio. The general geometric sequence is a, ar, ar^2, ar^3, $\ldots$, ar^{n-1}, where a is the first term, r is the common ratio, and n is the number of terms. Therefore, the formula for the nth term of a geometric sequence is $t_n = ar^{n-1}$.

The following example shows how the formulas can be used to find the nth term.

Example

Find the 12th term in the following sequences:

a. 5, 7, 9, 11, …

b. 5, 10, 20, 40, …

Solution

a. This is an arithmetic sequence.

$a=5$ and $d=2$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_{12} &= 5+(12-1)2 \\ &= 5+(11)2 \\ &= 5+22 \\ &= 27 \end{aligned}$$

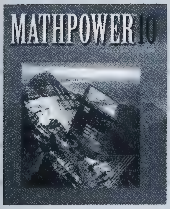
The 12th term is 27.

b. This is a geometric sequence.

$a=5$ and $r=2$

$$\begin{aligned} t_n &= ar^{n-1} \\ t_{12} &= 5\times 2^{12-1} \\ &= 5\times 2^{11} \\ &= 5\times 2048 \\ &= 10\,240 \end{aligned}$$

The 12th term is 10 240.



1. Turn to page 90 of *MATHPOWER™ 10* and answer questions 21 and 22 of "Review."

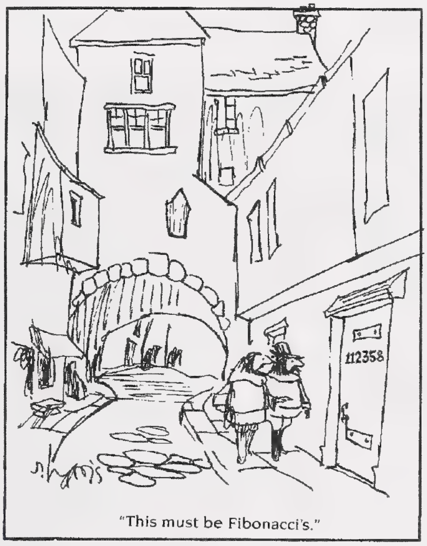
2. Find the indicated term for each geometric sequence.

   a. $1, 3, 9, 27, \ldots, t_8$ **b.** $2, 6, 18, 54, \ldots, t_{10}$

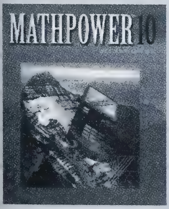
Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Extra Help.

Enrichment

In this section, you explored the Fibonacci sequence. Why do the characters in the following cartoon conclude that the house is Fibonacci's?



"This must be Fibonacci's."[1]



1. Turn to page 24 of *MATHPOWER™ 10* and answer question 63 of "Applications and Problem Solving."

Compare your response with the one in the Appendix, Section 2: Follow-up Activities, Enrichment.

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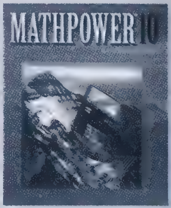
[1] Sidney Harris, cartoon, 1979. Reprinted by permission of Sidney Harris.

Many authors say that the formula you have just worked with was discovered in 1843 by mathematician Jacques Philippe Marie Binet; thus, the formula is called Binet's formula. However, some authors believe that mathematician Abraham de Moivre had written about this formula more than 100 years before Binet is supposed to have discovered it.

If you wish to find out more information on Fibonacci, Binet, and de Moivre, you can begin your research at the MacTutor History of Mathematics archive on the Internet at the following website:

http://www-groups.dcs.st-and.ac.uk/~history/index.html

Question 63 on page 24 of *MATHPOWER™ 10* states that many aspects of nature, including the number of pairs of rabbits in a family and the number of branches on a tree, can be described using the Fibonacci sequence.



For more information on the Fibonacci numbers and nature, visit the following website on the Internet:

http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html

The following number trick is based on the Fibonacci sequence.

Choose any two numbers, and write the numbers one below the other. Add the first and second numbers to get a third number, and write this number below the second number. Add the second and third numbers to get a fourth number, and write this number below the third number. Continue in this fashion until there are as many numbers as you wish. Then draw a line between any two numbers.

You can quickly find the sum of all the numbers above the line by simply subtracting the second number in the list from the second number below the line.

For example, if you choose 5 and 9, write a total of 12 numbers; then draw a line between the ninth and tenth numbers.

Numbers

5
9
14
23
37
60
97
157
254
411
665
1076

665 – 9 = 656

In this example, the sum of the first nine numbers is 656.

2. Try this number trick using another pair of numbers.

3. Let *a* and *b* represent the two numbers you chose in question 2, and continue the following pattern until there are eight numbers.

   Numbers

   a

   b

   $a+b$

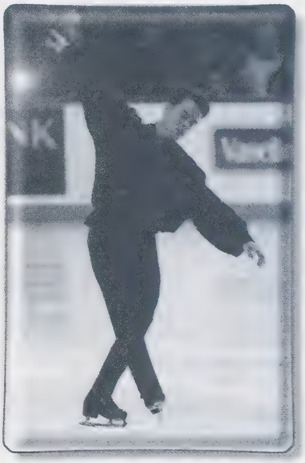
   $a+2b$

   Put a line between the sixth and seventh numbers. Use algebra to prove that the sum of the first six numbers is equal to the eighth number minus the second number.

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.

Conclusion

In this section you explored number patterns. You wrote formulas for generating the sequences, drew graphs of sequences, and calculated the sum of the terms of a sequence. You also used the sequence and sum functions on a graphing calculator.



CANADIAN FIGURE SKATING ASSOCIATION/
BOB BROOKS

In arithmetic sequences (such as 5, 7, 9, 11) and in geometric sequences (such as 2, 6, 18), there is a number pattern. There is also a pattern in the sequence of footwork for a figure-skating program.

In what other real situations are there sequences? Which of these examples have number patterns?

Assignment

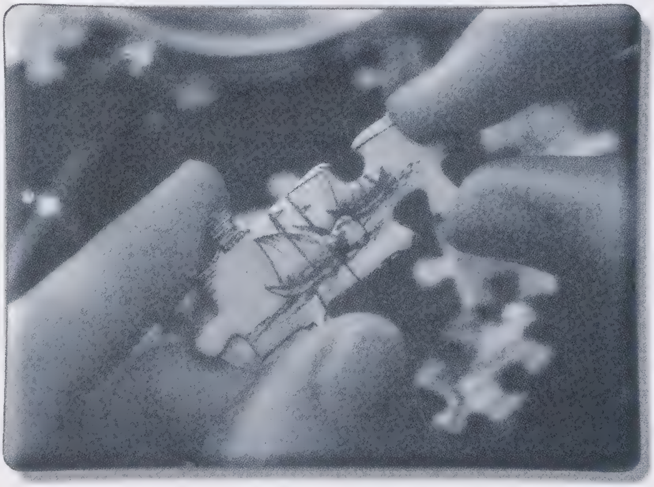
Turn to Assignment Booklet 4B and complete the assignment for Section 2.

Module Summary

In this module, you investigated number connections. In Section 1, you examined the real number system, performed operations on irrational numbers, and worked with powers that had rational exponents. In Section 2, you explored number patterns in sequences and series.

In this module, you discovered many connections. You discovered that performing operations on radicals is similar to performing operations on monomials and binomials in algebra. You found that you can use algebra to solve problems involving sequences and series.

Studying mathematics is similar to completing a jigsaw puzzle. If you pay attention to patterns and watch for connections, you will discover that the pieces of the puzzle begin to fit together, thus giving you a sense of the big picture.



Final Module Assignment

Turn to Assignment Booklet 4B and complete the final module assignment.

Appendix

Glossary

Suggested Answers

Glossary

arithmetic mean(s): the term(s) between two non-consecutive terms of an arithmetic sequence

arithmetic sequence: a sequence in which each successive term, after the first term, can be found by adding the same number

common difference: the difference between two consecutive terms of an arithmetic sequence

common ratio: the ratio of two consecutive terms in a geometric series

conjugate binomials: a pair of binomials that are the same except that the sign in the second is opposite the first

entire radical: a number written entirely underneath a root sign

finite sequence: a sequence that has both a first and last term

general term: the *n*th term of a sequence; the formula that relates a term to its position in the sequence

geometric mean(s): the term(s) between two non-consecutive terms of a geometric sequence

geometric sequence: a sequence in which each successive term, after the first term, can be found by multiplying by the same number

index: the number in the upper left-hand corner of the root sign that indicates which root is to be extracted

infinite sequence: a sequence that has no last term or no first term

irrational number: a real number that cannot be expressed as a ratio of two integers, a terminating decimal number, or a repeating decimal number

mixed radical: a number written with one part underneath a root sign and the other part outside the root sign

radical: a number that involves a root sign

radicand: the number or algebraic expression under the root sign

rationalizing the denominator: the process of eliminating a radical from the denominator of a fraction

rational number: a real number that can be expressed as the ratio of two integers, a terminating decimal number, or a repeating decimal number

real numbers: the set of all numbers on the number line

recursion formula: a formula in which a term of a sequence may be determined from the previous term

seed term(s): the first term(s) used to obtain successive terms

sequence: an ordered list of elements (things in the set such as numbers, letters, or diagrams)

series: the sum of the terms of a sequence

term: an element of a sequence

term number: the number that indicates the term's position in a sequence

Suggested Answers

Section 1: Activity 1

1. **a. Textbook questions 1 and 2 of Investigation 1, "One Digit Repeating," p. 4**

1. a. $\frac{7}{9} = 0.\overline{7}$ **b.** $\frac{7}{90} = 0.0\overline{7}$ **c.** $\frac{7}{900} = 0.00\overline{7}$

2. a. $0.\overline{5} = \frac{5}{9}$

b. $0.\overline{6} = \frac{6}{9} = \frac{2}{3}$

c. $0.0\overline{8} = \frac{8}{90} = \frac{4}{45}$

d. $0.00\overline{2} = \frac{2}{900} = \frac{1}{450}$

e. $0.00\overline{1} = \frac{1}{900}$

f. $1.\overline{3} = 1\frac{3}{9} = 1\frac{1}{3}$

g. $2.0\overline{4} = 2\frac{4}{90} = 2\frac{2}{45}$

h. $7.00\overline{2} = 7\frac{2}{900} = 7\frac{1}{450}$

b. Textbook questions 1 and 2 of Investigation 2, "Two Digits Repeating," p. 4

1. a. $\frac{47}{99} = 0.\overline{47}$ **b.** $\frac{47}{990} = 0.0\overline{47}$ **c.** $\frac{47}{9900} = 0.00\overline{47}$

2. a. $0.\overline{32} = \frac{32}{99}$

b. $0.\overline{63} = \frac{63}{99} = \frac{7}{11}$

c. $0.0\overline{45} = \frac{45}{990} = \frac{1}{22}$

d. $0.00\overline{56} = \frac{56}{9900} = \frac{14}{2475}$

e. $3.\overline{45} = 3\frac{45}{99} = 3\frac{5}{11}$

f. $6.\overline{66} = 6\frac{66}{99} = 6\frac{2}{3}$

Section 1: Activity 1 (continued)

g. $11.0\overline{35} = 11\frac{35}{990}$
$= 11\frac{7}{198}$

h. $22.00\overline{78} = 22\frac{78}{9900}$
$= 22\frac{13}{1650}$

c. **Textbook question 1 of Investigation 3, "Non-Repeating Parts," p. 4**

1. **a.** $0.3\overline{4} = 0.3 + 0.0\overline{4}$
$= \frac{3}{10} + \frac{4}{90}$
$= \frac{27}{90} + \frac{4}{90}$
$= \frac{31}{90}$

b. $0.5\overline{6} = 0.5 + 0.0\overline{6}$
$= \frac{5}{10} + \frac{6}{90}$
$= \frac{45}{90} + \frac{6}{90}$
$= \frac{51}{90}$
$= \frac{17}{30}$

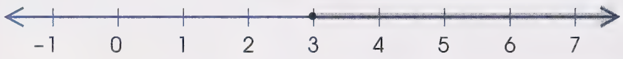
c. $0.1\overline{23} = 0.1 + 0.0\overline{23}$
$= \frac{1}{10} + \frac{23}{990}$
$= \frac{99}{990} + \frac{23}{990}$
$= \frac{122}{990}$
$= \frac{61}{495}$

d. $0.23\overline{4} = 0.23 + 0.00\overline{4}$
$= \frac{23}{100} + \frac{4}{900}$
$= \frac{207}{900} + \frac{4}{900}$
$= \frac{211}{900}$

e. $4.2\overline{5} = 4.2 + 0.0\overline{5}$
$= 4\frac{2}{10} + \frac{5}{90}$
$= 4\frac{18}{90} + \frac{5}{90}$
$= 4\frac{23}{90}$

2. **a.** **Textbook questions 21 to 26 of "Practice," p. 8**

21.



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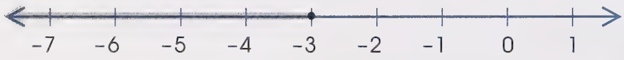

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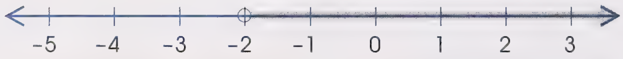

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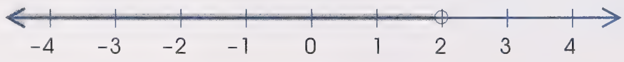

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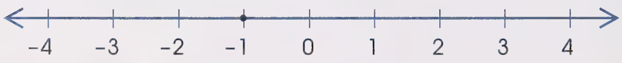
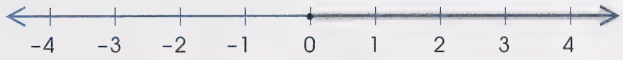
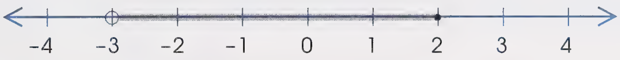


26.

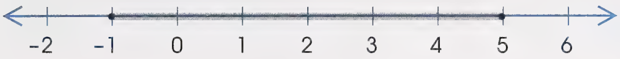


b. Textbook questions 27 to 30 of "Practice," p. 8

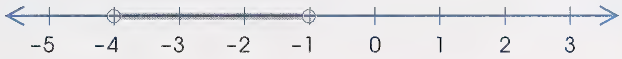
27.



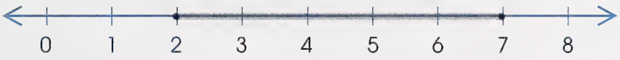
28.



29.



30.



c. Textbook questions 49 to 53 of "Practice," p. 8

49. $x > -3$ **50.** $x \le 2$ **51.** $-1 \le x \le 5$ **52.** $-3 < x < 0$ **53.** $-5 \le x < -1$

3. Textbook question "Explore: Complete the Table," p. 5

Rational Number	5	$1\frac{1}{3}$	0.75	$0.\overline{6}$	−1.04	$-0.\overline{34}$	0	$-2\frac{1}{2}$	$\sqrt{25}$
Form $\frac{a}{b}$	$\frac{5}{1}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{2}{3}$	$-\frac{26}{25}$	$-\frac{34}{99}$	$\frac{0}{1}$	$-\frac{5}{2}$	$\frac{5}{1}$

4. a. Textbook questions 1 to 3 of "Practice," p. 8

1. This is a repeating decimal number; therefore, it is rational.
2. This is a non-terminating, non-repeating decimal number; therefore, it is irrational.
3. This is a non-terminating, non-repeating decimal number; therefore, it is irrational.

b. Textbook questions 4 to 15 of "Practice," p. 8

4. This number is in the form $\frac{a}{b}$; therfore, it is rational.
5. This number is a terminating decimal; therefore, it is rational.
6. This number is a repeating decimal; therefore, it is rational.
7. This number is the square root of a non-perfect square; therefore, it is irrational.

Section 1: Activity 1 (continued)

8. $-\sqrt{225}=-15$

This number is equivalent to an integer; therefore, it is rational.

9. This number is the square root of a non-perfect square; therefore, it is irrational.

10. $-\sqrt{\frac{25}{49}}=-\frac{5}{7}$

This number is equivalent to a rational number; therefore, it is rational.

11. This number is the square root of a non-perfect square; therefore, it is irrational.

12. This number is a repeating decimal; therefore, it is rational.

13. $\sqrt{\sqrt{16}}=\sqrt{4}=2$

This number is equivalent to a natural number; therefore, it is rational.

14. $2\frac{1}{3}=\frac{7}{3}$

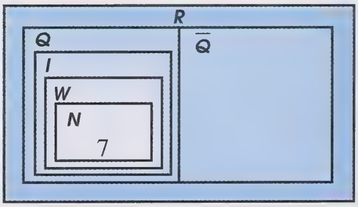
This number can be written in the form $\frac{a}{b}$; therefore, it is rational.

15. This is the symbol for a non-terminating, non-repeating decimal; therefore, it is irrational.

c. Textbook questions 16 to 19 of "Practice," p. 8

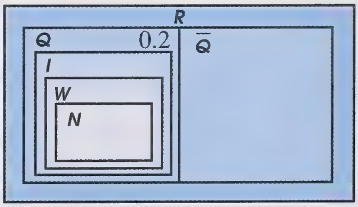
16. $\sqrt{49}=7$

This number is a natural number, whole number, integer, rational number, and real number.

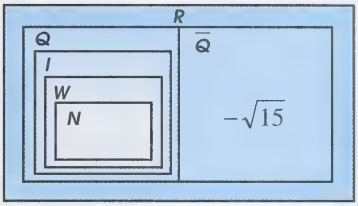


17. $\sqrt{0.04}=0.2$

This number is a rational number and a real number.

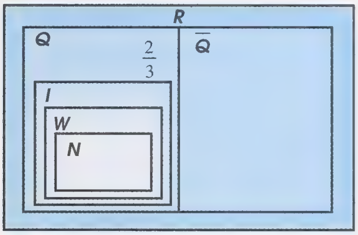


18. This is the negative square root of a non-perfect square; therefore, it is an irrational number and a real number.



19. $\sqrt{\frac{4}{9}} = \frac{2}{3}$

This number is a rational number and a real number.



d. **Textbook question 56 of "Applications and Problem Solving," p. 8**

56. The letter R was chosen to represent the real number system; therefore, a different letter was needed to represent rational numbers. Q was chosen because all rational numbers can be expressed in the form $\frac{a}{b}$, which is a **q**uotient.

5. a. **Textbook question 1 of "Practice," p. 14**

1. The approximation of 2.2 is less than $\sqrt{5}$.

The approximation of 2.24 is greater than $\sqrt{5}$.

The approximation of 2.236 is less than $\sqrt{5}$.

The approximation of 2.2361 is greater than $\sqrt{5}$.

b. **Textbook questions 2, 4, 6, 8, 10, and 12 of "Practice," p. 14**

	Estimate	**Calculated Approximate Value**
2.	$\sqrt{44} = \sqrt{\underset{\downarrow}{44}}$ 7	$\sqrt{44} \doteq 6.63$
4.	$\sqrt{792} = \sqrt{\underset{\downarrow}{07}\ \underset{\downarrow}{92}}$ $3\ \ 0$	$\sqrt{792} \doteq 28.14$

Section 1: Activity 1 (continued)

6. Estimate

$$\sqrt{23\,812} = \sqrt{02\,38\,12}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$1 \quad 0 \quad 0$$

Calculated Approximate Value

$$\sqrt{23\,812} \doteq 154.31$$

8. Estimate

$$\sqrt{5.06}$$
$$\downarrow \quad \downarrow$$
$$2. \quad 0$$

Calculated Approximate Value

$$\sqrt{5.06} \doteq 2.25$$

10. Estimate

$$\sqrt{0.045} = \sqrt{0.04\,50}$$
$$\downarrow \quad \downarrow$$
$$0. \quad 2$$

Calculated Approximate Value

$$\sqrt{0.045} \doteq 0.21$$

12. Estimate

$$\sqrt{0.000\,41} = \sqrt{0.00\,04\,10}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$0. \quad 0 \quad 2$$

Calculated Approximate Value

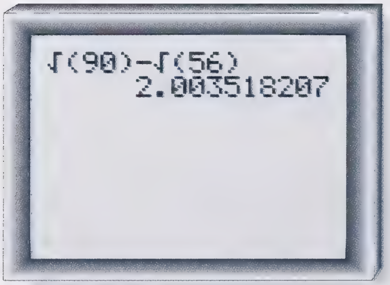
$$\sqrt{0.000\,41} \doteq 0.02$$

6. a. Textbook questions 15, 17, 19, and 21 of "Practice," p. 14

15. Estimate

$$\sqrt{90} - \sqrt{56} \doteq 10 - 8$$
$$\doteq 2$$

Calculated Approximate Value



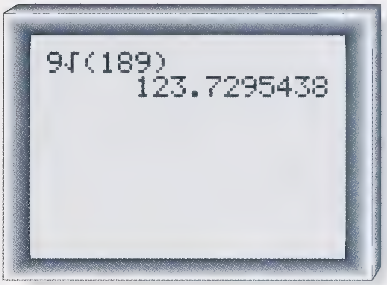
```
√(90)-√(56)
     2.003518207
```

$$\therefore \ \sqrt{90} - \sqrt{56} \doteq 2.0$$

17. Estimate

$$\begin{aligned} 9\sqrt{189} &\doteq 9\times 14 \\ &\doteq 126 \end{aligned}$$

Calculated Approximate Value



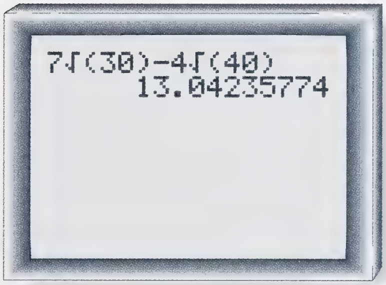
```
9√(189)
          123.7295438
```

$\therefore\ 9\sqrt{189} \doteq 123.7$

19. Estimate

$$\begin{aligned} 7\sqrt{30}-4\sqrt{40} &\doteq 7\times 5-4\times 6 \\ &\doteq 35-24 \\ &\doteq 11 \end{aligned}$$

Calculated Approximate Value



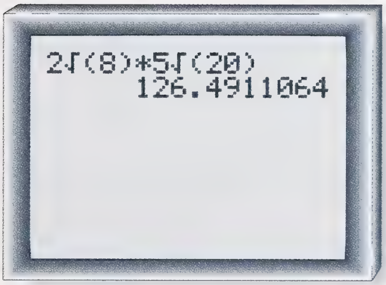
```
7√(30)-4√(40)
          13.04235774
```

$\therefore\ 7\sqrt{30}-4\sqrt{40} \doteq 13.0$

21. Estimate

$$\begin{aligned} 2\sqrt{8}\times 5\sqrt{20} &\doteq 2\times 3\times 5\times 4 \\ &\doteq 120 \end{aligned}$$

Calculated Approximate Value



```
2√(8)*5√(20)
          126.4911064
```

$\therefore\ 2\sqrt{8}\times 5\sqrt{20} \doteq 126.5$

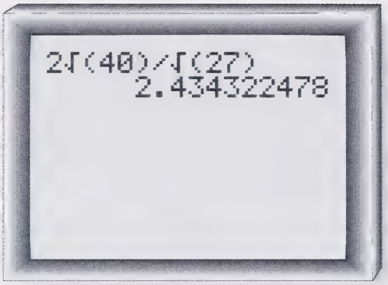
Section 1: Activity 1 (continued)

b. Textbook questions 23, 25, and 27 of "Practice," pp. 14 and 15

23. Estimate

$$\frac{2\sqrt{40}}{\sqrt{27}} \doteq \frac{2\times 6}{5}$$
$$\doteq \frac{12}{5}$$
$$\doteq 2.4$$

Calculated Approximate Value



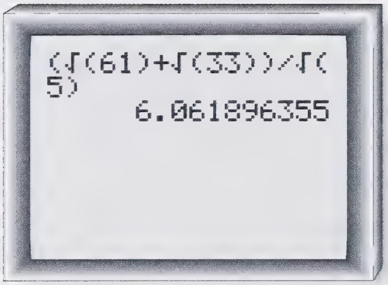
```
2√(40)/√(27)
         2.434322478
```

$$\therefore \frac{2\sqrt{40}}{\sqrt{27}} \doteq 2.4$$

25. Estimate

$$\frac{\sqrt{61}+\sqrt{33}}{\sqrt{5}} \doteq \frac{8+6}{2}$$
$$\doteq \frac{14}{2}$$
$$\doteq 7$$

Calculated Approximate Value



```
(√(61)+√(33))/√(
5)
         6.061896355
```

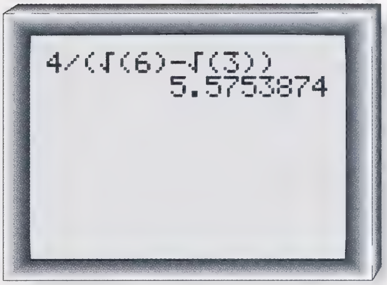
$$\therefore \frac{\sqrt{61}+\sqrt{33}}{\sqrt{5}} \doteq 6.1$$

27. Estimate

$$\frac{4}{\sqrt{6}-\sqrt{3}} \doteq \frac{4}{2-1}$$
$$\doteq \frac{4}{1}$$
$$\doteq 4$$

In order to avoid a 0 in the denominator, $\sqrt{3}$ was estimated to equal 1 rather than 2.

Calculated Approximate Value



```
4/(√(6)-√(3))
            5.5753874
```

$$\therefore \frac{4}{\sqrt{6}-\sqrt{3}} \doteq 5.6$$

7. Textbook questions 34 to 38 of "Practice," p. 15

34. $\sqrt[3]{8}+\sqrt[3]{27} = 2+3$
$= 5$

35. $\sqrt[3]{-27}+\sqrt[3]{125} = -3+5$
$= 2$

36. $2\left(\sqrt[3]{64}\right)+3\left(\sqrt[3]{-8}\right) = 2(4)+3(-2)$
$= 8-6$
$= 2$

37. $4\left(\sqrt[3]{-125}\right)-2\left(\sqrt[3]{-64}\right) = 4(-5)-2(-4)$
$= -20+8$
$= -12$

38. $\sqrt[4]{16}+\sqrt[4]{81} = 2+3$
$= 5$

8. Textbook questions 39 to 44 of "Practice," p. 15

39. $\sqrt[3]{35} \doteq 3.27$

40. $\sqrt[3]{90} \doteq 4.48$

41. $2\left(\sqrt[3]{22}\right) \doteq 5.60$

42. $4\left(\sqrt[3]{-30}\right) \doteq -12.43$

43. $\sqrt[3]{56}+\sqrt[3]{88} \doteq 8.27$

44. $3\left(\sqrt[3]{85}\right)+5\left(\sqrt[3]{22}\right) \doteq 27.20$

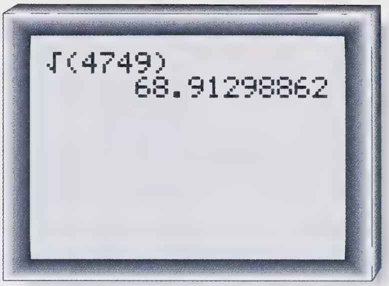
Section 1: Activity 1 (continued)

9. Textbook questions 48, 49, 51, 52, and 54.a. of "Applications and Problem Solving," pp. 15 and 16

48. a.

$$A = s^2$$
$$4749 = s^2$$
$$s \doteq 68.9$$

If the city had the shape of a square, each side would be about 68.9 km in length.



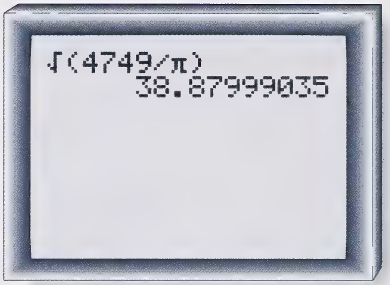
```
√(4749)
          68.91298862
```

b.

$$A = \pi r^2$$
$$4749 \doteq \pi\left(r^2\right)$$
$$\frac{4749}{\pi} \doteq r^2$$
$$r \doteq 38.9$$

$$\doteq 2(38.9)$$
$$\therefore\ d \doteq 77.8$$

If the city had the shape of a circle, its diameter would be about 77.8 km.

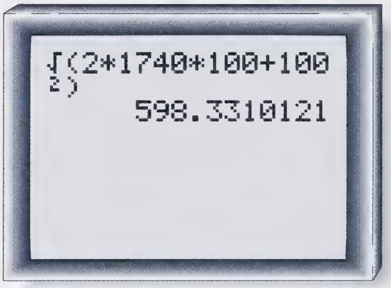


```
√(4749/π)
          38.87999035
```

49.

$$d = \sqrt{2rh + h^2}$$
$$= \sqrt{2(1740)(100) + 100^2}$$
$$\doteq 598$$

The distance to the horizon is about 598 km.

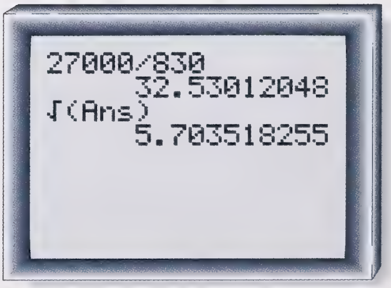


```
√(2*1740*100+100
²)
          598.3310121
```

51. a.

$$D^3 = 830t^2$$
$$30^3 = 830t^2$$
$$27\,000 = 830t^2$$
$$32.5 \doteq t^2$$
$$t \doteq 5.7$$

The storm is expected to last about 5.7 h.



```
27000/830
          32.53012048
√(Ans)
          5.703518255
```

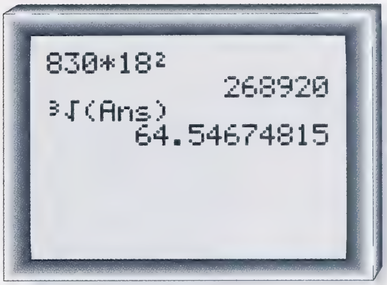
b. $D^3 = 830t^2$

$D^3 = 830(18)^2$

$D^3 = 268\,920$

$D \doteq 64.5$

The diameter is about 64.5 km.



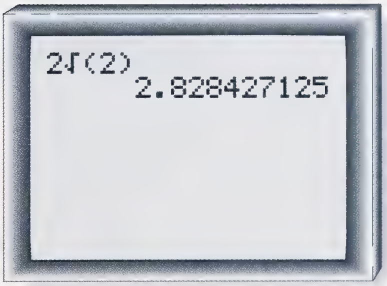
```
830*18²
             268920
³√(Ans)
        64.54674815
```

52. a. $T = 2\sqrt{\ell}$

$= 2\sqrt{2}$

$\doteq 2.8$

The 2-m pendulum takes about 2.8 s to swing back and forth once.



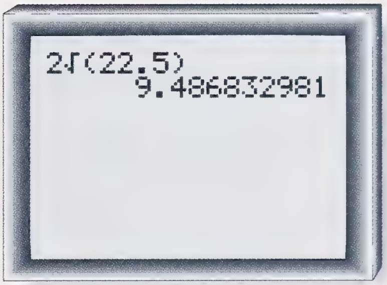
```
2√(2)
        2.828427125
```

b. $T = 2\sqrt{\ell}$

$= 2\sqrt{22.5}$

$\doteq 9.5$

The pendulum in the clock in Tokyo takes about 9.5 s to swing back and forth once.



```
2√(22.5)
        9.486832981
```

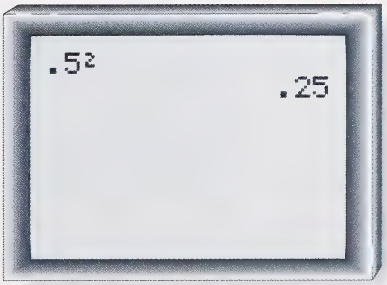
c. $T = 2\sqrt{\ell}$

$1 = 2\sqrt{\ell}$

$0.5 = \sqrt{\ell}$

$\ell = 0.25$

The pendulum is 0.25 m in length.



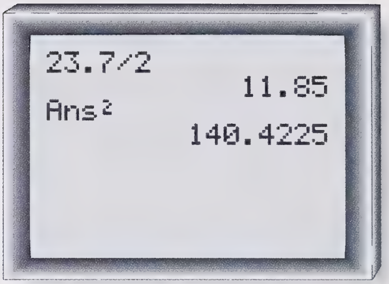
```
.5²
                .25
```

Section 1: Activity 1 (continued)

d.

$$\begin{aligned} T &= 2\sqrt{\ell} \\ 23.7 &= 2\sqrt{\ell} \\ 11.85 &= \sqrt{\ell} \\ \ell &\doteq 140 \end{aligned}$$

The Royal Centre Tower is about 140 m tall.

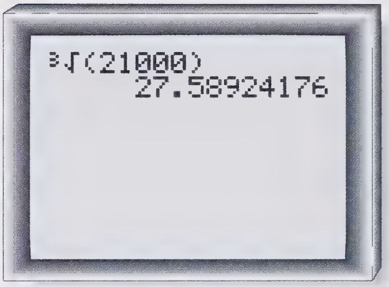


```
23.7/2
                11.85
Ans²
             140.4225
```

54. a.

$$\begin{aligned} V &= s^3 \\ 21\,000 &= s^3 \\ s &\doteq 27.6 \end{aligned}$$

If the nickel were made into a cube, the length of each edge would be about 27.6 m.



```
³√(21000)
          27.58924176
```

10. Textbook questions 7 and 9 of "Applications and Problem Solving," p. 11

7. Books are numbered so that the even-numbered pages appear on the left and the odd-numbered pages appear on the right. Since the last digit of 14 042 is a 2, the pages end in either 6 and 7 (since $6 \times 7 = 42$) or 8 and 9 (since $8 \times 9 = 72$). This helps to narrow the search.

The following chart shows several guesses and checks:

Even Page	Odd Page	Product	Is the Product Correct?
106	107	11 342	No, it's too low.
126	127	16 002	No, it's too high.
116	117	13 572	No, but it's close.
118	119	14 042	Yes!

Petra opened her math book to pages 118 and 119.

9. There are 4 wheels per car and 2 wheels per motorcycle.

The following chart shows several guesses and checks:

Cars	Motorcycles	Wheels	Is the Product Correct?
25	7	$25 \times 4 + 7 \times 2 = 114$	No, it's too high.
22	10	$22 \times 4 + 10 \times 2 = 108$	No, but it's close.
23	9	$23 \times 4 + 9 \times 2 = 110$	Yes!

There were 9 motorcycles on the dealership lot.

11. Answers may vary. Two sample answers have been given for each.

a. What is one plus, six divided by three?

or

What is one more than the quotient obtained by dividing six by three?

b. What is eight minus, two times three?

or

What is eight minus the product of two and three?

c. What is fourteen minus, one plus two?

or

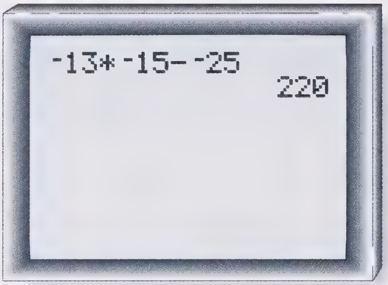
What is fourteen diminished by the sum of one and two?

Section 1: Activity 1 (continued)

12. **a.** Press the following keys on your graphing calculator:

(–) 1 3 × (–) 1 5 – (–) 2 5 ENTER

The calculator display will look like this:



```
-13*-15--25
            220
```

Do not confuse the negative sign, (–), and the subtraction key, –.

The answer is 220.

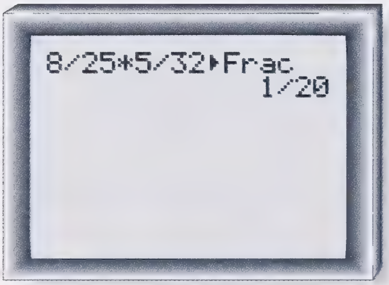
b. Press the following keys on your graphing calculator:

8 ÷ 2 5 × 5 ÷ 3 2

Then press MATH 1 to select the fraction function.

Finally, press ENTER.

The calculator display will look like this:



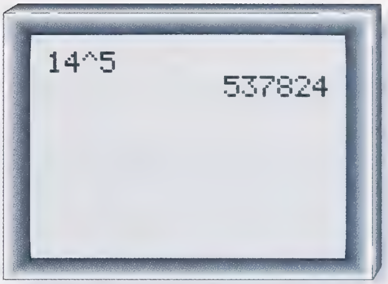
```
8/25*5/32▸Frac
          1/20
```

The answer is $\frac{1}{20}$.

c. Press the following keys:

1 4 ^ 5 ENTER

The calculator display looks like the one on the right.



```
14^5
                537824
```

Section 1: Activity 2

1. Textbook question 1 of "Using the Pythagorean Theorem," p. 2

1. a.

$$\begin{aligned} 1^2+1^2&=a^2\\ 1+1&=a^2\\ 2&=a^2\\ a&=\sqrt{2} \end{aligned}$$

$$\begin{aligned} a^2+1^2&=b^2\\ \left(\sqrt{2}\right)^2+1^2&=b^2\\ 2+1&=b^2\\ 3&=b^2\\ b&=\sqrt{3} \end{aligned}$$

$$\begin{aligned} b^2+1&=c^2\\ \left(\sqrt{3}\right)^2+1^2&=c^2\\ 3+1&=c^2\\ 4&=c^2\\ c&=\sqrt{4} \end{aligned}$$

$$\begin{aligned} c^2+1^2&=d^2\\ \left(\sqrt{4}\right)^2+1^2&=d^2\\ 4+1&=d^2\\ 5&=d^2\\ d&=\sqrt{5} \end{aligned}$$

$$\begin{aligned} d^2+1^2&=e^2\\ \left(\sqrt{5}\right)^2+1^2&=e^2\\ 5+1&=e^2\\ 6&=e^2\\ e&=\sqrt{6} \end{aligned}$$

$$\begin{aligned} e^2+1^2&=f^2\\ \left(\sqrt{6}\right)^2+1^2&=f^2\\ 6+1&=f^2\\ 7&=f^2\\ f&=\sqrt{7} \end{aligned}$$

$$\begin{aligned} f^2+1^2&=g^2\\ \left(\sqrt{7}\right)^2+1^2&=g^2\\ 7+1&=g^2\\ 8&=g^2\\ g&=\sqrt{8} \end{aligned}$$

b. If the pattern continues, $h=\sqrt{9}$, $i=\sqrt{10}$, and $j=\sqrt{11}$. Therefore, $k=\sqrt{12}$.

Section 1: Activity 2 (continued)

2. Textbook question "Explore: Complete the Table," p. 17

$\sqrt{4\times 9}=\sqrt{36}=6$	$\sqrt{4}\times\sqrt{9}=2\times 3=6$
$\sqrt{9\times 16}=\sqrt{144}=12$	$\sqrt{9}\times\sqrt{16}=3\times 4=12$
$\sqrt{25\times 4}=\sqrt{100}=10$	$\sqrt{25}\times\sqrt{4}=5\times 2=10$
$\sqrt{\frac{36}{4}}=\sqrt{9}=3$	$\frac{\sqrt{36}}{\sqrt{4}}=\frac{6}{2}=3$
$\sqrt{\frac{100}{25}}=\sqrt{4}=2$	$\frac{\sqrt{100}}{\sqrt{25}}=\frac{10}{5}=2$
$\sqrt{\frac{144}{9}}=\sqrt{16}=4$	$\frac{\sqrt{144}}{\sqrt{9}}=\frac{12}{3}=4$

3. Textbook questions 1, 3, 5, 7, 9, 11, 13, and 15 of "Practice," p. 19

1.
$$\begin{aligned}\sqrt{12}&=\sqrt{4\times 3}\\&=\sqrt{4}\times\sqrt{3}\\&=2\sqrt{3}\end{aligned}$$

3.
$$\begin{aligned}\sqrt{45}&=\sqrt{9\times 5}\\&=\sqrt{9}\times\sqrt{5}\\&=3\sqrt{5}\end{aligned}$$

5.
$$\begin{aligned}\sqrt{24}&=\sqrt{4\times 6}\\&=\sqrt{4}\times\sqrt{6}\\&=2\sqrt{6}\end{aligned}$$

7.
$$\begin{aligned}\sqrt{200}&=\sqrt{100\times 2}\\&=\sqrt{100}\times\sqrt{2}\\&=10\sqrt{2}\end{aligned}$$

9.
$$\begin{aligned}\sqrt{44}&=\sqrt{4\times 11}\\&=\sqrt{4}\times\sqrt{11}\\&=2\sqrt{11}\end{aligned}$$

11.
$$\begin{aligned}\sqrt{18}&=\sqrt{9\times 2}\\&=\sqrt{9}\times\sqrt{2}\\&=3\sqrt{2}\end{aligned}$$

13.
$$\begin{aligned}\sqrt{128}&=\sqrt{64\times 2}\\&=\sqrt{64}\times\sqrt{2}\\&=8\sqrt{2}\end{aligned}$$

15.
$$\begin{aligned}\sqrt{125}&=\sqrt{25\times 5}\\&=\sqrt{25}\times\sqrt{5}\\&=5\sqrt{5}\end{aligned}$$

4. a.
$$\begin{aligned}\sqrt{\frac{1}{4}}&=\frac{\sqrt{1}}{\sqrt{4}}\\&=\frac{1}{2}\end{aligned}$$

b.
$$\begin{aligned}\sqrt{\frac{5}{36}}&=\frac{\sqrt{5}}{\sqrt{36}}\\&=\frac{\sqrt{5}}{6}\end{aligned}$$

c.
$$\begin{aligned}\sqrt{\frac{7}{16}}&=\frac{\sqrt{7}}{\sqrt{16}}\\&=\frac{\sqrt{7}}{4}\end{aligned}$$

d.
$$\begin{aligned}\sqrt{\frac{2}{9}}&=\frac{\sqrt{2}}{\sqrt{9}}\\&=\frac{\sqrt{2}}{3}\end{aligned}$$

5. a. **Textbook questions 34 to 37 of "Practice," p. 19**

34. $$\begin{aligned}\sqrt{2}\times\sqrt{10} &= \sqrt{2\times 10}\\ &= \sqrt{20}\\ &= \sqrt{4\times 5}\\ &= \sqrt{4}\times\sqrt{5}\\ &= 2\sqrt{5}\end{aligned}$$

35. $$\begin{aligned}\sqrt{3}\times\sqrt{6} &= \sqrt{3\times 6}\\ &= \sqrt{18}\\ &= \sqrt{9\times 2}\\ &= \sqrt{9}\times\sqrt{2}\\ &= 3\sqrt{2}\end{aligned}$$

36. $$\begin{aligned}\sqrt{15}\times\sqrt{5} &= \sqrt{15\times 5}\\ &= \sqrt{75}\\ &= \sqrt{25\times 3}\\ &= \sqrt{25}\times\sqrt{3}\\ &= 5\sqrt{3}\end{aligned}$$

37. $$\begin{aligned}\sqrt{7}\times\sqrt{11} &= \sqrt{7\times 11}\\ &= \sqrt{77}\end{aligned}$$

b. **Textbook questions 16 to 20 of "Practice," p. 19**

16. $$\begin{aligned}\frac{\sqrt{14}}{\sqrt{7}} &= \sqrt{\frac{14}{7}}\\ &= \sqrt{2}\end{aligned}$$

17. $$\begin{aligned}\frac{\sqrt{10}}{\sqrt{2}} &= \sqrt{\frac{10}{2}}\\ &= \sqrt{5}\end{aligned}$$

18. $$\begin{aligned}\frac{\sqrt{60}}{\sqrt{3}} &= \sqrt{\frac{60}{3}}\\ &= \sqrt{20}\\ &= \sqrt{4\times 5}\\ &= \sqrt{4}\times\sqrt{5}\\ &= 2\sqrt{5}\end{aligned}$$

19. $$\begin{aligned}\frac{\sqrt{40}}{\sqrt{5}} &= \sqrt{\frac{40}{5}}\\ &= \sqrt{8}\\ &= \sqrt{4\times 2}\\ &= \sqrt{4}\times\sqrt{2}\\ &= 2\sqrt{2}\end{aligned}$$

20. $$\begin{aligned}\frac{\sqrt{33}}{\sqrt{3}} &= \sqrt{\frac{33}{3}}\\ &= \sqrt{11}\end{aligned}$$

Section 1: Activity 2 (continued)

6. a. Textbook questions 38, 41, and 42 of "Practice," p. 19

38. $$\begin{aligned}4\sqrt{3}\times\sqrt{7}&=4\times\sqrt{3}\times\sqrt{7}\\&=4\sqrt{21}\end{aligned}$$

41. $$\begin{aligned}2\sqrt{5}\times3\sqrt{10}&=2\times3\times\sqrt{5}\times\sqrt{10}\\&=6\sqrt{50}\\&=6\sqrt{25\times2}\\&=6\times\sqrt{25}\times\sqrt{2}\\&=6\times5\times\sqrt{2}\\&=30\sqrt{2}\end{aligned}$$

42. $$\begin{aligned}3\sqrt{3}\times4\sqrt{15}&=3\times4\times\sqrt{3}\times\sqrt{15}\\&=12\sqrt{45}\\&=12\sqrt{9\times5}\\&=12\times\sqrt{9}\times\sqrt{5}\\&=12\times3\times\sqrt{5}\\&=36\sqrt{5}\end{aligned}$$

b. Textbook questions 22 to 24 of "Practice," p. 19

22. $$\begin{aligned}\frac{3\sqrt{8}}{\sqrt{2}}&=3\times\sqrt{\frac{8}{2}}\\&=3\sqrt{4}\\&=3\times2\\&=6\end{aligned}$$

23. $$\begin{aligned}\frac{27\sqrt{15}}{3\sqrt{5}}&=\frac{27}{3}\times\sqrt{\frac{15}{5}}\\&=9\sqrt{3}\end{aligned}$$

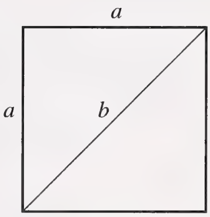
24. $$\begin{aligned}\frac{12\sqrt{75}}{4\sqrt{3}}&=\frac{12}{4}\times\sqrt{\frac{75}{3}}\\&=3\times\sqrt{25}\\&=3\times5\\&=15\end{aligned}$$

c. **Textbook questions 56 and 60 of "Applications and Problem Solving," p. 20**

56. $$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}\times\sqrt{6}\times\frac{\sqrt{10}}{2} \\ &= \frac{1}{2}\times\frac{1}{2}\times\sqrt{6}\times\sqrt{10} \\ &= \frac{1}{4}\times\sqrt{60} \\ &= \frac{1}{4}\times\sqrt{4\times15} \\ &= \frac{1}{4}\times\sqrt{4}\times\sqrt{15} \\ &= \frac{1}{4}\times2\times\sqrt{15} \\ &= \frac{\sqrt{15}}{2} \end{aligned}$$

The area is $\frac{\sqrt{15}}{2}$ square units.

60. a. Step 1: Find the length of each side of the board.
Hint: The board is a square.



Let *a* be the length of each side of the board.
Let *b* be the length of the diagonal of the board.

$$\begin{aligned} a^2 + a^2 &= b^2 \\ 2a^2 &= b^2 \\ 2a^2 &= \left(\sqrt{800}\right)^2 \\ 2a^2 &= 800 \\ a^2 &= 400 \\ a &= 20 \end{aligned}$$

Each side is 20 cm in length.

Section 1: Activity 2 (continued)

Step 2: Find the number of small squares on the board.

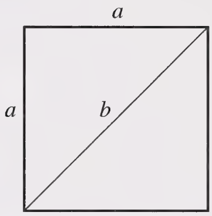
$$20 \div 2 = 10$$

The board is 10 squares by 10 squares.

Therefore, there are $10 \times 10 = 100$ squares on the board.

b. **Step 1:** Find the length of each side of the board.

Let a be the length of each side of the board.
Let b be the length of the diagonal of the board.



$$\begin{aligned} a^2 + a^2 &= b^2 \\ 2a^2 &= b^2 \\ 2a^2 &= \left(\sqrt{2592}\right)^2 \\ 2a^2 &= 2592 \\ a^2 &= 1296 \\ a &= 36 \end{aligned}$$

Each side is 36 cm in length.

Step 2: Find the number of squares on the board.

$$36 \div 3 = 12$$

The board is 12 squares by 12 squares.

Therefore, there are $12 \times 12 = 144$ squares on the board.

7. a. Textbook questions 1, 3, 5, and 7 of "Practice," p. 23

1. $2\sqrt{5} + 3\sqrt{5} + 6\sqrt{5} = 11\sqrt{5}$

3. $$\begin{aligned} 6\sqrt{2} - \sqrt{2} + 7\sqrt{2} - 3\sqrt{2} &= 6\sqrt{2} + 7\sqrt{2} - \sqrt{2} - 3\sqrt{2} \\ &= 13\sqrt{2} - 4\sqrt{2} \\ &= 9\sqrt{2} \end{aligned}$$

5. $8\sqrt{10}-2\sqrt{10}-7\sqrt{10}=8\sqrt{10}-9\sqrt{10}$

$$=-\sqrt{10}$$

7. $\sqrt{5}+\sqrt{5}+\sqrt{5}+\sqrt{5}=4\sqrt{5}$

b. Textbook questions 8, 10, 12, and 14 of "Practice," p. 23

8. $5\sqrt{3}+2\sqrt{6}+3\sqrt{3}=5\sqrt{3}+3\sqrt{3}+2\sqrt{6}$

$$=8\sqrt{3}+2\sqrt{6}$$

10. $2\sqrt{2}+3\sqrt{10}+5\sqrt{2}-4\sqrt{10}=2\sqrt{2}+5\sqrt{2}+3\sqrt{10}-4\sqrt{10}$

$$=7\sqrt{2}-\sqrt{10}$$

12. $9\sqrt{11}-\sqrt{11}+6\sqrt{14}-3\sqrt{14}-2\sqrt{11}=9\sqrt{11}-\sqrt{11}-2\sqrt{11}+6\sqrt{14}-3\sqrt{14}$

$$=6\sqrt{11}+3\sqrt{14}$$

14. $8+7\sqrt{11}-9-9\sqrt{11}=8-9+7\sqrt{11}-9\sqrt{11}$

$$=-1-2\sqrt{11}$$

c. Textbook question 71 of "Applications and Problem Solving," p. 24

71. If either $a=0$ or $b=0$, the equation is true.
For example, if $a=0$ and $b=1$, then

$$\sqrt{a+b}=\sqrt{a}+\sqrt{b}$$
$$\sqrt{0+1}=\sqrt{0}+\sqrt{1}$$
$$\sqrt{1}=\sqrt{1} \quad \longleftarrow \text{This is a true statement.}$$

If $a>0$ and $b>0$, the equation is not true.
For example, if $a=1$ and $b=1$, then

$$\sqrt{a+b}=\sqrt{a}+\sqrt{b}$$
$$\sqrt{1+1}=\sqrt{1}+\sqrt{1}$$
$$\sqrt{2}=1+1$$
$$\sqrt{2}=2 \quad \longleftarrow \text{This is not a true statement.}$$

Section 1: Activity 2 (continued)

8. a. Textbook questions 15, 17, 19, and 21 of "Practice," p. 23

15.
$$\begin{aligned}\sqrt{12}+\sqrt{27}&=\sqrt{4\times3}+\sqrt{9\times3}\\&=\sqrt{4}\times\sqrt{3}+\sqrt{9}\times\sqrt{3}\\&=2\sqrt{3}+3\sqrt{3}\\&=5\sqrt{3}\end{aligned}$$

17.
$$\begin{aligned}\sqrt{18}-\sqrt{8}&=\sqrt{9\times2}-\sqrt{4\times2}\\&=\sqrt{9}\times\sqrt{2}-\sqrt{4}\times\sqrt{2}\\&=3\sqrt{2}-2\sqrt{2}\\&=\sqrt{2}\end{aligned}$$

19.
$$\begin{aligned}\sqrt{75}+\sqrt{48}+\sqrt{27}&=\sqrt{25\times3}+\sqrt{16\times3}+\sqrt{9\times3}\\&=\sqrt{25}\times\sqrt{3}+\sqrt{16}\times\sqrt{3}+\sqrt{9}\times\sqrt{3}\\&=5\sqrt{3}+4\sqrt{3}+3\sqrt{3}\\&=12\sqrt{3}\end{aligned}$$

21.
$$\begin{aligned}\sqrt{28}-\sqrt{27}+\sqrt{63}+\sqrt{300}&=\sqrt{4\times7}-\sqrt{9\times3}+\sqrt{9\times7}+\sqrt{100\times3}\\&=\sqrt{4}\times\sqrt{7}-\sqrt{9}\times\sqrt{3}+\sqrt{9}\times\sqrt{7}+\sqrt{100}\times\sqrt{3}\\&=2\sqrt{7}-3\sqrt{3}+3\sqrt{7}+10\sqrt{3}\\&=2\sqrt{7}+3\sqrt{7}-3\sqrt{3}+10\sqrt{3}\\&=5\sqrt{7}+7\sqrt{3}\end{aligned}$$

b. Textbook questions 22, 24, 26, and 28 of "Practice," p. 23

22.
$$\begin{aligned}8\sqrt{7}+2\sqrt{28}&=8\sqrt{7}+2\sqrt{4\times7}\\&=8\sqrt{7}+2\times\sqrt{4}\times\sqrt{7}\\&=8\sqrt{7}+2\times2\times\sqrt{7}\\&=8\sqrt{7}+4\sqrt{7}\\&=12\sqrt{7}\end{aligned}$$

24. $$\begin{aligned}5\sqrt{27}+4\sqrt{48}&=5\sqrt{9\times3}+4\sqrt{16\times3}\\&=5\times\sqrt{9}\times\sqrt{3}+4\times\sqrt{16}\times\sqrt{3}\\&=5\times3\times\sqrt{3}+4\times4\times\sqrt{3}\\&=15\sqrt{3}+16\sqrt{3}\\&=31\sqrt{3}\end{aligned}$$

26. $$\begin{aligned}\sqrt{5}+2\sqrt{45}-3\sqrt{20}&=\sqrt{5}+2\sqrt{9\times5}+3\sqrt{4\times5}\\&=\sqrt{5}+2\times\sqrt{9}\times\sqrt{5}-3\times\sqrt{4}\times\sqrt{5}\\&=\sqrt{5}+2\times3\times\sqrt{5}-3\times2\times\sqrt{5}\\&=\sqrt{5}+6\sqrt{5}-6\sqrt{5}\\&=\sqrt{5}\end{aligned}$$

28. $$\begin{aligned}3\sqrt{48}-4\sqrt{8}+4\sqrt{27}-2\sqrt{72}&=3\sqrt{16\times3}-4\sqrt{4\times2}+4\sqrt{9\times3}-2\sqrt{36\times2}\\&=3\times\sqrt{16}\times\sqrt{3}-4\times\sqrt{4}\times\sqrt{2}+4\times\sqrt{9}\times\sqrt{3}-2\times\sqrt{36}\times\sqrt{2}\\&=3\times4\times\sqrt{3}-4\times2\times\sqrt{2}+4\times3\times\sqrt{3}-2\times6\times\sqrt{2}\\&=12\sqrt{3}-8\sqrt{2}+12\sqrt{3}-12\sqrt{2}\\&=12\sqrt{3}+12\sqrt{3}-8\sqrt{2}-12\sqrt{2}\\&=24\sqrt{3}-20\sqrt{2}\end{aligned}$$

c. Textbook question 66 of "Applications and Problem Solving," p. 24

66. $$\begin{aligned}\sqrt{5}+\sqrt{20}+\sqrt{45}+\sqrt{80}&=\sqrt{5}+\sqrt{4\times5}+\sqrt{9\times5}+\sqrt{16\times5}\\&=\sqrt{5}+\sqrt{4}\times\sqrt{5}+\sqrt{9}\times\sqrt{5}+\sqrt{16}\times\sqrt{5}\\&=\sqrt{5}+2\sqrt{5}+3\sqrt{5}+4\sqrt{5}\\&=10\sqrt{5}\end{aligned}$$

The perimeter is $10\sqrt{5}$ units.

Section 1: Activity 2 (continued)

9. a. Textbook questions 29 to 34 of “Practice,” p. 23

29. $$\begin{aligned}\sqrt{2}\left(\sqrt{10}+4\right)&=\sqrt{2}\times\sqrt{10}+\sqrt{2}\times 4\\&=\sqrt{20}+4\sqrt{2}\\&=\sqrt{4\times 5}+4\sqrt{2}\\&=\sqrt{4}\times\sqrt{5}+4\sqrt{2}\\&=2\sqrt{5}+4\sqrt{2}\end{aligned}$$

30. $$\begin{aligned}\sqrt{3}\left(\sqrt{6}-1\right)&=\sqrt{3}\times\sqrt{6}+\sqrt{3}\times(-1)\\&=\sqrt{18}-\sqrt{3}\\&=\sqrt{9\times 2}-\sqrt{3}\\&=\sqrt{9}\times\sqrt{2}-\sqrt{3}\\&=3\sqrt{2}-\sqrt{3}\end{aligned}$$

31. $$\begin{aligned}\sqrt{6}\left(\sqrt{2}+\sqrt{6}\right)&=\sqrt{6}\times\sqrt{2}+\sqrt{6}\times\sqrt{6}\\&=\sqrt{12}+6\\&=\sqrt{4\times 3}+6\\&=\sqrt{4}\times\sqrt{3}+6\\&=2\sqrt{3}+6\end{aligned}$$

32. $$\begin{aligned}2\sqrt{2}\left(3\sqrt{6}-\sqrt{3}\right)&=2\sqrt{2}\times 3\sqrt{6}+2\sqrt{2}\times\left(-\sqrt{3}\right)\\&=2\times 3\times\sqrt{2}\times\sqrt{6}+2\times(-1)\times\sqrt{2}\times\sqrt{3}\\&=6\sqrt{12}-2\sqrt{6}\\&=6\times\sqrt{4\times 3}-2\sqrt{6}\\&=6\times\sqrt{4}\times\sqrt{3}-2\sqrt{6}\\&=6\times 2\times\sqrt{3}-2\sqrt{6}\\&=12\sqrt{3}-2\sqrt{6}\end{aligned}$$

33. $$\begin{aligned}\sqrt{2}\left(\sqrt{3}+4\right)&=\sqrt{2}\times\sqrt{3}+\sqrt{2}\times 4\\&=\sqrt{6}+4\sqrt{2}\end{aligned}$$

34. $$\begin{aligned}3\sqrt{2}\left(2\sqrt{6}+\sqrt{10}\right)&=3\sqrt{2}\times 2\sqrt{6}+3\sqrt{2}\times\sqrt{10}\\&=3\times 2\times\sqrt{2}\times\sqrt{6}+3\times\sqrt{2}\times\sqrt{10}\\&=6\sqrt{12}+3\sqrt{20}\\&=6\times\sqrt{4\times 3}+3\times\sqrt{4\times 5}\\&=6\times\sqrt{4}\times\sqrt{3}+3\times\sqrt{4}\times\sqrt{5}\\&=6\times 2\times\sqrt{3}+3\times 2\times\sqrt{5}\\&=12\sqrt{3}+6\sqrt{5}\end{aligned}$$

b. Textbook question 64 of "Applications and Problem Solving," p. 24

64. a. $$\begin{aligned}A&=\ell w\\&=2\sqrt{3}\left(4\sqrt{2}-\sqrt{3}\right)\\&=2\sqrt{3}\times 4\sqrt{2}+2\sqrt{3}\times\left(-\sqrt{3}\right)\\&=2\times 4\times\sqrt{3}\times\sqrt{2}+2\times(-1)\times\sqrt{3}\times\sqrt{3}\\&=8\sqrt{6}-2\times 3\\&=8\sqrt{6}-6\end{aligned}$$

The area of the rectangle is $8\sqrt{6}-6$ square units.

b. $$\begin{aligned}P&=2\ell+2w\\&=2\left(2\sqrt{3}\right)+2\left(4\sqrt{2}-\sqrt{3}\right)\\&=2\times 2\times\sqrt{3}+2\times 4\times\sqrt{2}+2\times\left(-\sqrt{3}\right)\\&=4\sqrt{3}+8\sqrt{2}-2\sqrt{3}\\&=4\sqrt{3}-2\sqrt{3}+8\sqrt{2}\\&=2\sqrt{3}+8\sqrt{2}\end{aligned}$$

The perimeter of the rectangle is $2\sqrt{3}+8\sqrt{2}$ units.

10. a. Textbook questions 35 to 39 of "Practice," p. 23

35. $$\begin{aligned}\left(\sqrt{5}+\sqrt{6}\right)\left(\sqrt{5}+3\sqrt{6}\right)&=\sqrt{5}\times\sqrt{5}+\sqrt{5}\times 3\sqrt{6}+\sqrt{6}\times\sqrt{5}+\sqrt{6}\times 3\sqrt{6}\\&=5+3\sqrt{30}+\sqrt{30}+3\times 6\\&=5+4\sqrt{30}+18\\&=23+4\sqrt{30}\end{aligned}$$

Section 1: Activity 2 (continued)

36. $$\begin{aligned}\left(2\sqrt{3}-1\right)\left(3\sqrt{3}+2\right)&=2\sqrt{3}\times3\sqrt{3}+2\sqrt{3}\times2-1\times3\sqrt{3}-1\times2\\&=6\times3+4\sqrt{3}-3\sqrt{3}-2\\&=18+\sqrt{3}-2\\&=16+\sqrt{3}\end{aligned}$$

37. $$\begin{aligned}\left(4\sqrt{7}-3\sqrt{2}\right)\left(2\sqrt{7}+5\sqrt{2}\right)&=4\sqrt{7}\times2\sqrt{7}+4\sqrt{7}\times5\sqrt{2}-3\sqrt{2}\times2\sqrt{7}-3\sqrt{2}\times5\sqrt{2}\\&=8\times7+20\sqrt{14}-6\sqrt{14}-15\times2\\&=56+14\sqrt{14}-30\\&=26+14\sqrt{14}\end{aligned}$$

38. $$\begin{aligned}\left(3\sqrt{3}+1\right)^2&=\left(3\sqrt{3}+1\right)\left(3\sqrt{3}+1\right)\\&=3\sqrt{3}\times3\sqrt{3}+3\sqrt{3}\times1+1\times3\sqrt{3}+1\times1\\&=9\times3+3\sqrt{3}+3\sqrt{3}+1\\&=27+6\sqrt{3}+1\\&=28+6\sqrt{3}\end{aligned}$$

39. $$\begin{aligned}\left(2\sqrt{2}-\sqrt{5}\right)^2&=\left(2\sqrt{2}-\sqrt{5}\right)\left(2\sqrt{2}-\sqrt{5}\right)\\&=2\sqrt{2}\times2\sqrt{2}+2\sqrt{2}\times\left(-\sqrt{5}\right)-\sqrt{5}\times2\sqrt{2}-\sqrt{5}\times\left(-\sqrt{5}\right)\\&=4\times2-2\sqrt{10}-2\sqrt{10}+5\\&=8-4\sqrt{10}+5\\&=13-4\sqrt{10}\end{aligned}$$

b. Textbook question 65 of "Applications and Problem Solving," p. 24

65.
$$\begin{aligned}A &= s^2\\ &= \left(\sqrt{8}-\sqrt{5}\right)^2\\ &= \left(\sqrt{8}-\sqrt{5}\right)\left(\sqrt{8}-\sqrt{5}\right)\\ &= \sqrt{8}\times\sqrt{8}+\sqrt{8}\times\left(-\sqrt{5}\right)-\sqrt{5}\times\sqrt{8}-\sqrt{5}\times\left(-\sqrt{5}\right)\\ &= 8-\sqrt{40}-\sqrt{40}+5\\ &= 13-2\sqrt{40}\\ &= 13-2\times\sqrt{4\times10}\\ &= 13-2\times\sqrt{4}\times\sqrt{10}\\ &= 13-2\times2\times\sqrt{10}\\ &= 13-4\sqrt{10}\end{aligned}$$

The area of the square is $13-4\sqrt{10}$ square units.

11. a. Textbook questions 40 to 42 of "Practice," p. 23

40.
$$\begin{aligned}\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) &= 4-3\\ &= 1\end{aligned}$$

41.
$$\begin{aligned}\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right) &= 6-2\\ &= 4\end{aligned}$$

42.
$$\begin{aligned}\left(2\sqrt{7}+3\sqrt{5}\right)\left(2\sqrt{7}-3\sqrt{5}\right) &= 4\times7-9\times5\\ &= 28-45\\ &= -17\end{aligned}$$

b. Textbook question 67 of "Practice," p. 24

67.
$$\begin{aligned}V &= \ell wh\\ &= \left(\sqrt{15}-\sqrt{2}\right)\left(5\sqrt{2}+2\sqrt{3}\right)\left(5\sqrt{2}-2\sqrt{3}\right) \longleftarrow \text{Multiply the conjugates first.}\\ &= \left(\sqrt{15}-\sqrt{2}\right)\left(25\times2-4\times3\right)\\ &= \left(\sqrt{15}-\sqrt{2}\right)\left(50-12\right)\\ &= \left(\sqrt{15}-\sqrt{2}\right)\left(38\right)\\ &= 38\sqrt{15}-38\sqrt{2}\end{aligned}$$

The volume of the rectangular prism is $38\sqrt{15}-38\sqrt{2}$ cubic units.

Section 1: Activity 2 (continued)

12. a. Textbook questions 46, 48, 50, 52, and 54 of "Practice," p. 20

46.
$$\sqrt{\frac{1}{3}}=\frac{\sqrt{1}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{\sqrt{3}}{3}$$

48.
$$\sqrt{\frac{5}{6}}=\frac{\sqrt{5}}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$
$$=\frac{\sqrt{30}}{6}$$

50.
$$\frac{2\sqrt{2}}{\sqrt{18}}=\frac{2\sqrt{2}}{\sqrt{18}}\times\frac{\sqrt{18}}{\sqrt{18}}$$
$$=\frac{2\sqrt{36}}{18}$$
$$=\frac{2\times 6}{18}$$
$$=\frac{12}{18}$$
$$=\frac{2}{3}$$

or

$$\frac{2\sqrt{2}}{\sqrt{18}}=\frac{2\sqrt{2}}{\sqrt{9\times 2}} \longleftarrow \text{Simplify.}$$
$$=\frac{2\sqrt{2}}{\sqrt{9}\times\sqrt{2}}$$
$$=\frac{2\sqrt{2}}{3\sqrt{2}}$$
$$=\frac{2}{3}$$

52.
$$\frac{4\sqrt{2}}{\sqrt{8}}=\frac{4\sqrt{2}}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}}$$
$$=\frac{4\sqrt{16}}{8}$$
$$=\frac{4\times 4}{8}$$
$$=\frac{16}{8}$$
$$=2$$

or

$$\frac{4\sqrt{2}}{\sqrt{8}}=\frac{4\sqrt{2}}{\sqrt{4\times 2}} \longleftarrow \text{Simplify.}$$
$$=\frac{4\sqrt{2}}{\sqrt{4}\times\sqrt{2}}$$
$$=\frac{4\sqrt{2}}{2\sqrt{2}}$$
$$=2$$

54. $$\frac{3\sqrt{6}}{4\sqrt{10}}=\frac{3\sqrt{6}}{4\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}}$$
$$=\frac{3\sqrt{60}}{4\times10}$$
$$=\frac{3\times\sqrt{4\times15}}{40}$$
$$=\frac{3\times\sqrt{4}\times\sqrt{15}}{40}$$
$$=\frac{3\times2\times\sqrt{15}}{40}$$
$$=\frac{6\sqrt{15}}{40}$$
$$=\frac{3\sqrt{15}}{20}$$

b. Textbook question 61 of "Applications and Problem Solving," p. 20

61. $$\frac{1}{\sqrt[3]{2}}=\frac{1}{\sqrt[3]{2}}\times\frac{\sqrt[3]{2}\times\sqrt[3]{2}}{\sqrt[3]{2}\times\sqrt[3]{2}}$$
$$=\frac{\sqrt[3]{4}}{2}$$

13. a. Textbook questions 44, 46, 48, 50, and 52 of "Practice," p. 23

44. $$\frac{3}{\sqrt{5}-1}=\frac{3}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$$
$$=\frac{3\sqrt{5}+3}{5-1}$$
$$=\frac{3\sqrt{5}+3}{4}$$

46. $$\frac{2}{\sqrt{6}+\sqrt{3}}=\frac{2}{\sqrt{6}+\sqrt{3}}\times\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}}$$
$$=\frac{2\sqrt{6}-2\sqrt{3}}{6-3}$$
$$=\frac{2\sqrt{6}-2\sqrt{3}}{3}$$

48. $$\frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$
$$=\frac{3-\sqrt{6}}{3-2}$$
$$=\frac{3-\sqrt{6}}{1}$$
$$=3-\sqrt{6}$$

50. $$\frac{\sqrt{2}-1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$$
$$=\frac{2-\sqrt{2}-\sqrt{2}+1}{2-1}$$
$$=\frac{2-2\sqrt{2}+1}{1}$$
$$=3-2\sqrt{2}$$

Section 1: Activity 2 (continued)

52.
$$\begin{aligned}\frac{2\sqrt{7}+\sqrt{5}}{3\sqrt{7}-2\sqrt{5}} &= \frac{2\sqrt{7}+\sqrt{5}}{3\sqrt{7}-2\sqrt{5}}\times\frac{3\sqrt{7}+2\sqrt{5}}{3\sqrt{7}+2\sqrt{5}}\\ &= \frac{6\times7+4\sqrt{35}+3\sqrt{35}+2\times5}{9\times7-4\times5}\\ &= \frac{42+7\sqrt{35}+10}{63-20}\\ &= \frac{52+7\sqrt{35}}{43}\end{aligned}$$

b. Textbook questions 68 and 69 of "Applications and Problem Solving," p. 24

68.
$$\begin{aligned}A &= \ell w\\ 4 &= \ell\left(\sqrt{7}-\sqrt{5}\right)\\ \frac{4}{\sqrt{7}-\sqrt{5}} &= \ell\\ \ell &= \frac{4}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}\\ &= \frac{4\sqrt{7}+4\sqrt{5}}{7-5}\\ &= \frac{4\sqrt{7}+4\sqrt{5}}{2}\\ &= 2\sqrt{7}+2\sqrt{5}\end{aligned}$$

The length of the rectangle is $2\sqrt{7}+2\sqrt{5}$ units.

69. Step 1: Calculate the areas of both circles in terms of π.

Area of Larger Circle

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(2+\sqrt{3}\right)^2 \\ &= \pi\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right) \\ &= \pi\left(4+2\sqrt{3}+2\sqrt{3}+3\right) \\ &= \pi\left(7+4\sqrt{3}\right) \end{aligned}$$

Area of Smaller Circle

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(2-\sqrt{3}\right)^2 \\ &= \pi\left(2-\sqrt{3}\right)\left(2-\sqrt{3}\right) \\ &= \pi\left(4-2\sqrt{3}-2\sqrt{3}+3\right) \\ &= \pi\left(7-4\sqrt{3}\right) \end{aligned}$$

Step 2: Calculate the ratio.

$$\begin{aligned} \frac{\text{Area of Larger Circle}}{\text{Area of Smaller Circle}} &= \frac{\overset{1}{\cancel{\pi}}\left(7+4\sqrt{3}\right)}{\underset{1}{\cancel{\pi}}\left(7-4\sqrt{3}\right)} \\ &= \frac{7+4\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} \\ &= \frac{49+28\sqrt{3}+28\sqrt{3}+16\times 3}{49-16\times 3} \\ &= \frac{49+56\sqrt{3}+48}{49-48} \\ &= \frac{97+56\sqrt{3}}{1} \\ &= 97+56\sqrt{3} \end{aligned}$$

The ratio of the area of the larger circle to the area of the smaller circle is $97+56\sqrt{3}$ to 1.

Section 1: Activity 2 (continued)

14. Textbook questions 43 and 45 of “Practice,” p. 23

43.
$$\begin{aligned}\frac{1}{\sqrt{2}+2} &= \frac{1}{\sqrt{2}+2}\times\frac{\sqrt{2}-2}{\sqrt{2}-2}\\ &= \frac{\sqrt{2}-2}{2-4}\\ &= \frac{\sqrt{2}-2}{-2}\\ &= \frac{\sqrt{2}-2}{-2}\times\frac{-1}{-1}\\ &= \frac{-\sqrt{2}+2}{2}\text{ or }\frac{2-\sqrt{2}}{2}\end{aligned}$$

45.
$$\begin{aligned}\frac{\sqrt{2}}{\sqrt{6}-3} &= \frac{\sqrt{2}}{\sqrt{6}-3}\times\frac{\sqrt{6}+3}{\sqrt{6}+3}\\ &= \frac{\sqrt{12}+3\sqrt{2}}{6-9}\\ &= \frac{\sqrt{4\times 3}+3\sqrt{2}}{-3}\\ &= \frac{\sqrt{4}\times\sqrt{3}+3\sqrt{2}}{-3}\\ &= \frac{2\sqrt{3}+3\sqrt{2}}{-3}\\ &= \frac{2\sqrt{3}+3\sqrt{2}}{-3}\times\frac{-1}{-1}\\ &= \frac{-2\sqrt{3}-3\sqrt{2}}{3}\end{aligned}$$

15. a. Textbook questions 25, 27, and 29 of “Practice,” p. 19

25. $$\begin{aligned}2\sqrt{3} &= \sqrt{4}\times\sqrt{3}\\ &= \sqrt{12}\end{aligned}$$

27. $$\begin{aligned}3\sqrt{10} &= \sqrt{9}\times\sqrt{10}\\ &= \sqrt{90}\end{aligned}$$

29. $$\begin{aligned}2\sqrt{7} &= \sqrt{4}\times\sqrt{7}\\ &= \sqrt{28}\end{aligned}$$

b. Textbook questions 31 to 33 of “Practice,” p. 19

31. Step 1: Write the mixed radicals as entire radicals.

$$\begin{aligned}3\sqrt{5} &= \sqrt{9}\times\sqrt{5}\\ &= \sqrt{45}\end{aligned}$$

$$\begin{aligned}2\sqrt{11} &= \sqrt{4}\times\sqrt{11}\\ &= \sqrt{44}\end{aligned}$$

$$\begin{aligned}4\sqrt{3} &= \sqrt{16}\times\sqrt{3}\\ &= \sqrt{48}\end{aligned}$$

$$\begin{aligned}5\sqrt{2} &= \sqrt{25}\times\sqrt{2}\\ &= \sqrt{50}\end{aligned}$$

Step 2: Compare the entire radicals.

$$\sqrt{44} < \sqrt{45} < \sqrt{48} < \sqrt{50}$$
$$\therefore\ 2\sqrt{11} < 3\sqrt{5} < 4\sqrt{3} < 5\sqrt{2}$$

The radicals listed in order from least to greatest are $2\sqrt{11}$, $3\sqrt{5}$, $4\sqrt{3}$, and $5\sqrt{2}$.

32. **Step 1:** Write the mixed radicals as entire radicals.

$$\begin{aligned} 6\sqrt{2} &= \sqrt{36} \times \sqrt{2} \\ &= \sqrt{72} \end{aligned} \qquad \begin{aligned} 3\sqrt{7} &= \sqrt{9} \times \sqrt{7} \\ &= \sqrt{63} \end{aligned}$$

$$8 = \sqrt{64} \qquad \begin{aligned} 2\sqrt{15} &= \sqrt{4} \times \sqrt{15} \\ &= \sqrt{60} \end{aligned}$$

Step 2: Compare the entire radicals.

$$\sqrt{60} < \sqrt{63} < \sqrt{64} < \sqrt{72}$$
$$\therefore\ 2\sqrt{15} < 3\sqrt{7} < 8 < 6\sqrt{2}$$

The radicals listed in order from least to greatest are $2\sqrt{15}$, $3\sqrt{7}$, 8, and $6\sqrt{2}$.

33. **Step 1:** Write the mixed radicals as entire radicals.

$$\begin{aligned} 5\sqrt{5} &= \sqrt{25} \times \sqrt{5} \\ &= \sqrt{125} \end{aligned} \qquad \begin{aligned} 4\sqrt{7} &= \sqrt{16} \times \sqrt{7} \\ &= \sqrt{112} \end{aligned}$$

$$\begin{aligned} 3\sqrt{14} &= \sqrt{9} \times \sqrt{14} \\ &= \sqrt{126} \end{aligned} \qquad \begin{aligned} 2\sqrt{30} &= \sqrt{4} \times \sqrt{30} \\ &= \sqrt{120} \end{aligned}$$

Step 2: Compare the entire radicals.

$$\sqrt{112} < \sqrt{120} < \sqrt{125} < \sqrt{126}$$
$$\therefore\ 4\sqrt{7} < 2\sqrt{30} < 5\sqrt{5} < 3\sqrt{14}$$

The radicals listed in order from least to greatest are $4\sqrt{7}$, $2\sqrt{30}$, $5\sqrt{5}$, and $3\sqrt{14}$.

Section 1: Activity 2 (continued)

16. **Step 1:** Find the decimal approximation of each radical expression.

$$\begin{aligned}\sqrt{7}+2 &\doteq 2.646+2\\ &\doteq 4.646\end{aligned} \qquad \begin{aligned}\sqrt{3}+3 &\doteq 1.732+3\\ &\doteq 4.732\end{aligned}$$

$$\begin{aligned}\sqrt{26}-1 &\doteq 5.099-1\\ &\doteq 4.099\end{aligned} \qquad \begin{aligned}6-\sqrt{3} &\doteq 6-1.732\\ &\doteq 4.268\end{aligned}$$

Step 2: Compare the decimal approximations.

$$\begin{aligned}&4.099 < 4.268 < 4.646 < 4.732\\ \therefore\ &\sqrt{26}-1 < 6-\sqrt{3} < \sqrt{7}+2 < \sqrt{3}+3\end{aligned}$$

The expressions listed in order from least to greatest are $\sqrt{26}-1$, $6-\sqrt{3}$, $\sqrt{7}+2$, and $\sqrt{3}+3$.

17. **Textbook questions 61 and 62 of "Applications and Problem Solving," pp. 23 and 24**

61. **Step 1:** Simplify each radical expression.

$$\sqrt{3}\left(\sqrt{3}+1\right) = 3+\sqrt{3} \qquad \begin{aligned}\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) &= 3-1\\ &= 2\end{aligned}$$

$$\begin{aligned}\left(1-\sqrt{3}\right)^2 &= 1-2\sqrt{3}+3\\ &= 4-2\sqrt{3}\end{aligned} \qquad \begin{aligned}\left(\sqrt{3}+1\right)^2 &= 3+2\sqrt{3}+1\\ &= 4+2\sqrt{3}\end{aligned}$$

Step 2: Estimate the decimal value of each radical expression.

$$\begin{aligned}\sqrt{3}\left(\sqrt{3}+1\right) &= 3+\sqrt{3}\\ &\doteq 3+2\\ &\doteq 5\end{aligned} \qquad \left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) = 2$$

$$\begin{aligned}\left(1-\sqrt{3}\right)^2 &= 4-2\sqrt{3}\\ &\doteq 4-2\times 2\\ &\doteq 0\end{aligned} \qquad \begin{aligned}\left(\sqrt{3}+1\right)^2 &= 4+2\sqrt{3}\\ &\doteq 4+2\times 2\\ &\doteq 4+4\\ &\doteq 8\end{aligned}$$

Step 3: Compare the estimated values.

$$8>5>2>0$$

$$\therefore\ \left(\sqrt{3}+1\right)^2>\sqrt{3}\left(\sqrt{3}+1\right)>\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)>\left(1-\sqrt{3}\right)^2$$

The expressions listed in order from greatest to least are $\left(\sqrt{3}+1\right)^2$, $\sqrt{3}\left(\sqrt{3}+1\right)$, $\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)$, and $\left(1-\sqrt{3}\right)^2$.

62. **a.** Simplify each radical expression.

$$\begin{aligned}\frac{60}{\sqrt{450}}&=\frac{60}{\sqrt{9}\times\sqrt{25}\times\sqrt{2}}\\&=\frac{60}{3\times5\times\sqrt{2}}\\&=\frac{60}{15\sqrt{2}}\\&=\frac{4}{\sqrt{2}}\\&=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{4\sqrt{2}}{2}\\&=2\sqrt{2}\end{aligned}$$

$$\begin{aligned}\frac{8}{\sqrt{18}}+\frac{4}{\sqrt{18}}&=\frac{12}{\sqrt{18}}\\&=\frac{12}{\sqrt{9}\times\sqrt{2}}\\&=\frac{12}{3\sqrt{2}}\\&=\frac{4}{\sqrt{2}}\\&=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{4\sqrt{2}}{2}\\&=2\sqrt{2}\end{aligned}$$

$$6\sqrt{2}-4\sqrt{2}=2\sqrt{2}$$

$$\begin{aligned}\frac{4}{\sqrt{2}}&=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{4\sqrt{2}}{2}\\&=2\sqrt{2}\end{aligned}$$

$$\begin{aligned}6\sqrt{8}+\sqrt{8}-5\sqrt{8}&=2\sqrt{8}\\&=2\times\sqrt{4}\times\sqrt{2}\\&=2\times2\times\sqrt{2}\\&=4\sqrt{2}\end{aligned}$$

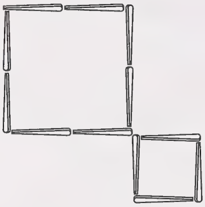
All the expressions equal $2\sqrt{2}$ except for $6\sqrt{8}+\sqrt{8}-5\sqrt{8}$.

b. The radical expression $6\sqrt{8}+\sqrt{8}-5\sqrt{8}$ is twice the others $\left(4\sqrt{2}\right)$.

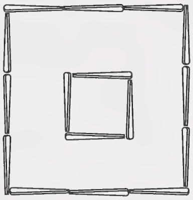
Section 1: Activity 2 (continued)

18. a. Textbook questions 1 and 2 of "LOGIC POWER," p. 9

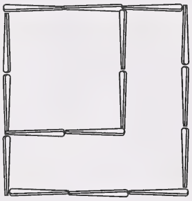
1.



2. a.

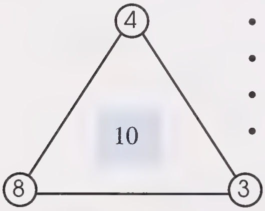


b.



b. Textbook question "PATTERN POWER," p. 16

The number in the centre of each diagram is the sum of the differences of each pair of numbers in the diagram.



- For the first diagram, $5-3=\mathbf{2}$, $9-3=\mathbf{6}$, $9-5=\mathbf{4}$, and $\mathbf{2}+\mathbf{6}+\mathbf{4}=12$.
- For the second diagram, $8-7=\mathbf{1}$, $7-1=\mathbf{6}$, $8-1=\mathbf{7}$, and $\mathbf{1}+\mathbf{6}+\mathbf{7}=14$.
- For the third diagram, $9-6=\mathbf{3}$, $6-2=\mathbf{4}$, $9-2=\mathbf{7}$, and $\mathbf{3}+\mathbf{4}+\mathbf{7}=14$.
- For the fourth diagram, $8-4=\mathbf{4}$, $8-3=\mathbf{5}$, $4-3=\mathbf{1}$, and $\mathbf{4}+\mathbf{5}+\mathbf{1}=10$.

c. Textbook question "LOGIC POWER," p. 24

Answers may vary. Two sample answers are given.

B	A	C	D
D	E	B	A
A	C	D	E
E	B	A	C

D	B	A	E
A	C	D	B
C	A	E	D
B	E	C	A

Section 1: Activity 3

1. a. **Textbook questions 13 to 16 of "Practice," p. 33**

13. $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

14. $5^0 = 1$

15. $2^{-3} = \frac{1}{2^3}$

$= \frac{1}{8}$

16. $(-2)^{-4} = \frac{1}{(-2)^4}$

$= \frac{1}{16}$

b. **Textbook questions 19 and 20 of "Practice," p. 33**

19.
$$\begin{aligned}\frac{1}{5^{-2}} &= \frac{1}{\frac{1}{5^2}}\\ &= \frac{1}{\frac{1}{25}}\\ &= 1 \div \frac{1}{25}\\ &= 1 \times \frac{25}{1}\\ &= 25\end{aligned}$$

20.
$$\begin{aligned}\frac{1}{(-4)^{-1}} &= \frac{1}{\frac{1}{(-4)^1}}\\ &= \frac{1}{-\frac{1}{4}}\\ &= 1 \div \left(-\frac{1}{4}\right)\\ &= 1 \times \left(\frac{-4}{1}\right)\\ &= -4\end{aligned}$$

2. a.
$$\begin{aligned}\left(\frac{1}{5}\right)^{-2} &= \frac{1}{\left(\frac{1}{5}\right)^2}\\ &= \frac{1}{\frac{1}{25}}\\ &= 1 \div \frac{1}{25}\\ &= 1 \times \frac{25}{1}\\ &= 25\end{aligned}$$

b.
$$\begin{aligned}\left(\frac{8}{9}\right)^{-1} &= \frac{1}{\frac{8}{9}}\\ &= 1 \div \frac{8}{9}\\ &= 1 \times \frac{9}{8}\\ &= \frac{9}{8} \text{ or } 1\frac{1}{8}\end{aligned}$$

c. $\left(-\frac{3}{7}\right)^0 = 1$

d. $\left(\frac{11}{12}\right)^0 = 1$

Section 1: Activity 3 (continued)

3. a. Textbook questions 1 to 4, 9, and 11 of "Practice," p. 33

1. $$\begin{aligned} 2^4 \times 2^3 &= 2^{4+3} \\ &= 2^7 \end{aligned}$$

2. $$\begin{aligned} 2^6 \div 2^2 &= 2^{6-2} \\ &= 2^4 \end{aligned}$$

3. $$\begin{aligned} \left(2^4\right)^3 &= 2^{3\times4} \\ &= 2^{12} \end{aligned}$$

4. $$\begin{aligned} 2 \times 2^7 &= 2^{1+7} \\ &= 2^8 \end{aligned}$$

9. $$\begin{aligned} 2^{-3} \times 2^4 &= 2^{-3+4} \\ &= 2^1 \text{ or } 2 \end{aligned}$$

11. $$\begin{aligned} \left(2^3\right)^{-1} &= 2^{3\times(-1)} \\ &= 2^{-3} \end{aligned}$$

b. Textbook question 84 of "Applications and Problem Solving," p. 33

84. Step 1: Write the powers with the same base.

$$20^{100} \qquad \begin{aligned} 400^{40} &= \left[(20)^2\right]^{40} \\ &= 20^{80} \end{aligned}$$

Step 2: Compare the powers.

$$20^{100} > 20^{80}$$

$$\therefore\ 20^{100} > 400^{40}$$

Therefore, the power 20^{100} is bigger.

4. a.

x	1	2	3	4	5	6	7	8	9
$x^{\frac{1}{2}}$	1	1.41	1.73	2	2.24	2.45	2.65	2.83	3

b. $x^{\frac{1}{2}}$ is equal to $\sqrt{x}$.

5. a.

x	1	2	3	4	5	6	7	8	9
$x^{\frac{1}{3}}$	1	1.26	1.44	1.59	1.71	1.82	1.91	2	2.08

b. $x^{\frac{1}{3}}$ is equal to $\sqrt[3]{x}$.

6. **a. Textbook questions 1 to 4 of "Explore: Complete the Statements," p. 34**

1. $9=\left(9^{\frac{1}{2}}\right)^2$

but $9=(3)^2$

so $\left(9^{\frac{1}{2}}\right)^2=(3)^2$

and $9^{\frac{1}{2}}=3$

2. $25=\left(25^{\frac{1}{2}}\right)^2$

but $25=(5)^2$

so $\left(25^{\frac{1}{2}}\right)^2=(5)^2$

and $25^{\frac{1}{2}}=5$

3. $8=\left(8^{\frac{1}{3}}\right)^3$

but $8=(2)^3$

so $\left(8^{\frac{1}{3}}\right)^3=(2)^3$

and $8^{\frac{1}{3}}=2$

4. $16=\left(16^{\frac{1}{4}}\right)^4$

but $16=(2)^4$

so $\left(16^{\frac{1}{4}}\right)^4=(2)^4$

and $16^{\frac{1}{4}}=2$

b. Textbook questions 1 to 3 of "Inquire," p. 35

1. $25^{\frac{1}{2}}=\sqrt{25}$

2. a. $8^{\frac{1}{3}}=\sqrt[3]{8}$ **b.** $16^{\frac{1}{4}}=\sqrt[4]{16}$

3. a. $36^{\frac{1}{2}}=\sqrt{36}$
$=6$

b. $27^{\frac{1}{3}}=\sqrt[3]{27}$
$=3$

c. $81^{\frac{1}{4}}=\sqrt[4]{81}$
$=3$

d. $100^{\frac{1}{2}}=\sqrt{100}$
$=10$

7. **a. Textbook questions 1, 7, and 8 of "Practice," p. 37**

1. $2^{\frac{1}{3}}=\sqrt[3]{2}$

7. $7^{-\frac{1}{2}}=\dfrac{1}{7^{\frac{1}{2}}}$
$=\dfrac{1}{\sqrt{7}}$

8. $9^{-\frac{1}{5}}=\dfrac{1}{9^{\frac{1}{5}}}$
$=\dfrac{1}{\sqrt[5]{9}}$

b. Textbook questions 13 to 15 of "Practice," p. 37

13. $\sqrt{7}=7^{\frac{1}{2}}$ **14.** $\sqrt{34}=34^{\frac{1}{2}}$ **15.** $\sqrt[3]{-11}=(-11)^{\frac{1}{3}}$

Section 1: Activity 3 (continued)

c. **Textbook questions 25 to 36 of "Practice," p. 37**

25. $$\begin{aligned}4^{\frac{1}{2}} &= \sqrt{4}\\ &= 2\end{aligned}$$

26. $$\begin{aligned}125^{\frac{1}{3}} &= \sqrt[3]{125}\\ &= 5\end{aligned}$$

27. $$\begin{aligned}16^{-\frac{1}{4}} &= \frac{1}{16^{\frac{1}{4}}}\\ &= \frac{1}{\sqrt[4]{16}}\\ &= \frac{1}{2}\end{aligned}$$

28. $$\begin{aligned}(-32)^{\frac{1}{5}} &= \sqrt[5]{-32}\\ &= -2\end{aligned}$$

29. $$\begin{aligned}25^{0.5} &= 25^{\frac{1}{2}}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

30. $$\begin{aligned}(-27)^{-\frac{1}{3}} &= \frac{1}{(-27)^{\frac{1}{3}}}\\ &= \frac{1}{\sqrt[3]{-27}}\\ &= \frac{1}{-3}\\ &= -\frac{1}{3}\end{aligned}$$

31. $$\begin{aligned}(64)^{-\frac{1}{6}} &= \frac{1}{(64)^{\frac{1}{6}}}\\ &= \frac{1}{\sqrt[6]{64}}\\ &= \frac{1}{2}\end{aligned}$$

32. $$\begin{aligned}0.04^{\frac{1}{2}} &= \sqrt{0.04}\\ &= 0.2\end{aligned}$$

33. $$\begin{aligned}81^{0.25} &= 81^{\frac{1}{4}}\\ &= \sqrt[4]{81}\\ &= 3\end{aligned}$$

34. $$\begin{aligned}0.001^{\frac{1}{3}} &= \sqrt[3]{0.001}\\ &= 0.1\end{aligned}$$

35. $$\begin{aligned}\left(\frac{4}{9}\right)^{\frac{1}{2}} &= \sqrt{\frac{4}{9}}\\ &= \frac{2}{3}\end{aligned}$$

36. $$\begin{aligned}\left(\frac{-27}{-8}\right)^{\frac{1}{3}} &= \left(\frac{27}{8}\right)^{\frac{1}{3}}\\ &= \sqrt[3]{\frac{27}{8}}\\ &= \frac{3}{2}\end{aligned}$$

8. Textbook questions 2, 5, and 6 of "Practice," p. 37

2. Method 1

$$37^{\frac{3}{2}} = \left(37^{\frac{1}{2}}\right)^3 = \left(\sqrt{37}\right)^3$$

Method 2

$$37^{\frac{3}{2}} = \left(37^3\right)^{\frac{1}{2}} = \sqrt{37^3}$$

5. Method 1

$$6^{\frac{4}{3}} = \left(6^{\frac{1}{3}}\right)^4 = \left(\sqrt[3]{6}\right)^4$$

Method 2

$$6^{\frac{4}{3}} = \left(6^4\right)^{\frac{1}{3}} = \sqrt[3]{6^4}$$

6. Method 1

$$6^{\frac{3}{4}} = \left(6^{\frac{1}{4}}\right)^3 = \left(\sqrt[4]{6}\right)^3$$

Method 2

$$6^{\frac{3}{4}} = \left(6^3\right)^{\frac{1}{4}} = \sqrt[4]{6^3}$$

9. a. $\sqrt{5^3} = 5^{\frac{3}{2}}$ **b.** $\left(\sqrt{7}\right)^5 = 7^{\frac{5}{2}}$

c. $\sqrt[3]{2^4} = 2^{\frac{4}{3}}$ **d.** $\left(\sqrt[4]{3}\right)^5 = 3^{\frac{5}{4}}$

10. Textbook questions 37, 39, 41, 43, 45, and 47 of "Practice," p. 37

37. $$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$$

39. $$9^{2.5} = 9^{\frac{5}{2}} = \left(9^{\frac{1}{2}}\right)^5 = \left(\sqrt{9}\right)^5 = 3^5 = 243$$

41. $$16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}} = \frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{(2)^3} = \frac{1}{8}$$

Section 1: Activity 3 (continued)

43. $$\begin{aligned}(-8)^{-\frac{5}{3}} &= \frac{1}{(-8)^{\frac{5}{3}}}\\ &= \frac{1}{\left[(-8)^{\frac{1}{3}}\right]^5}\\ &= \frac{1}{\left(\sqrt[3]{-8}\right)^5}\\ &= \frac{1}{(-2)^5}\\ &= \frac{1}{-32}\\ &= -\frac{1}{32}\end{aligned}$$

45. $$\begin{aligned}1^{\frac{5}{3}} &= \left(1^{\frac{1}{3}}\right)^5\\ &= \left(\sqrt[3]{1}\right)^5\\ &= 1^5\\ &= 1\end{aligned}$$

47. $$\begin{aligned}\left(\frac{100}{9}\right)^{\frac{3}{2}} &= \left[\left(\frac{100}{9}\right)^{\frac{1}{2}}\right]^3\\ &= \left(\sqrt{\frac{100}{9}}\right)^3\\ &= \left(\frac{10}{3}\right)^3\\ &= \frac{1000}{27}\end{aligned}$$

11. Textbook questions 52, 54, 56, and 64 of "Practice," p. 37

52. $$\begin{aligned}3^{\frac{1}{2}} \times 3^{\frac{1}{2}} &= 3^{\frac{1}{2}+\frac{1}{2}}\\ &= 3^{\frac{2}{2}}\\ &= 3^1\\ &= 3\end{aligned}$$

54. $$\begin{aligned}\left(2^5\right)^{0.4} &= 2^{5\times 0.4}\\ &= 2^2\\ &= 4\end{aligned}$$

56. $$\begin{aligned}4^{\frac{3}{2}} \div 16^{\frac{1}{4}} &= 4^{\frac{3}{2}} \div \left(4^2\right)^{\frac{1}{4}}\\ &= 4^{\frac{3}{2}} \div 4^{2\times\frac{1}{4}}\\ &= 4^{\frac{3}{2}} \div 4^{\frac{1}{2}}\\ &= 4^{\frac{3}{2}-\frac{1}{2}}\\ &= 4^{\frac{2}{2}}\\ &= 4^1\\ &= 4\end{aligned}$$

64. $$\begin{aligned}9^{\frac{3}{7}} \times 3^{\frac{1}{7}} &= \left(3^2\right)^{\frac{3}{7}} \times 3^{\frac{1}{7}}\\ &= 3^{2\times\frac{3}{7}} \times 3^{\frac{1}{7}}\\ &= 3^{\frac{6}{7}} \times 3^{\frac{1}{7}}\\ &= 3^{\frac{6}{7}+\frac{1}{7}}\\ &= 3^{\frac{7}{7}}\\ &= 3^1\\ &= 3\end{aligned}$$

12. Textbook questions 66 and 67 of "Practice," p. 37

66. $\sqrt[3]{\sqrt{64}} = \sqrt[3]{8}$

$= 2$

67. $\sqrt{\sqrt[3]{729}} = \sqrt{9}$

$= 3$

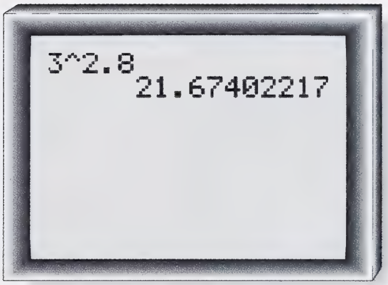
13. Textbook questions 85, 87, and 89 of "Practice," p. 38

85. Estimate

$3^{2.8} \doteq 3^3$

$\doteq 27$

Calculated Approximate Value



```
3^2.8
          21.67402217
```

$\therefore\ 3^{2.8} \doteq 21.67$

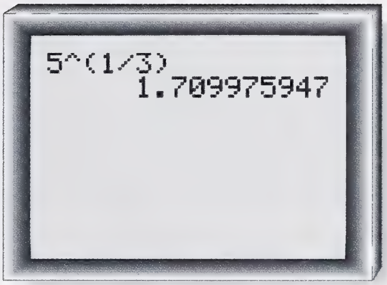
87. Estimate

$5^{\frac{1}{3}} \doteq 8^{\frac{1}{3}}$

$\doteq \sqrt[3]{8}$

$\doteq 2$

Calculated Approximate Value



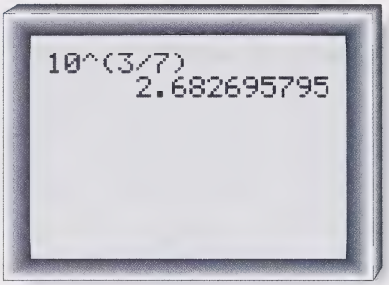
```
5^(1/3)
          1.709975947
```

$\therefore\ 5^{\frac{1}{3}} \doteq 1.71$

Section 1: Activity 3 (continued)

89. Estimate **Calculated Approximate Value**

$$10^{\frac{3}{7}} \doteq 10^{\frac{1}{2}}$$
$$\doteq 9^{\frac{1}{2}}$$
$$\doteq \sqrt{9}$$
$$\doteq 3$$



```
10^(3/7)
         2.682695795
```

$\therefore\ 10^{\frac{3}{7}} \doteq 2.68$

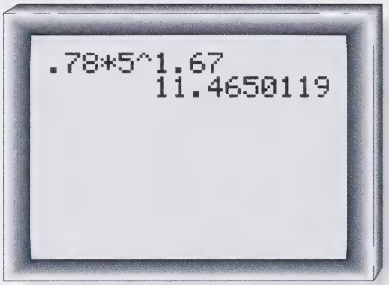
14. a. Textbook questions 1 and 2 of Investigation 1, "Calculating Speeds," p. 44

1. $s = \dfrac{0.78\ell^{1.67}}{h^{1.17}}$

$= \dfrac{0.78 \times 5^{1.67}}{1^{1.17}}$

$= 0.78 \times 5^{1.67}$ $1^{1.17} = 1$

$\doteq 11.5$



```
.78*5^1.67
          11.4650119
```

Bailey's speed was about 11.5 m/s.

2. Step 1: Calculate the hip length of the theropod.

$$4 \times 0.64 = 2.56$$

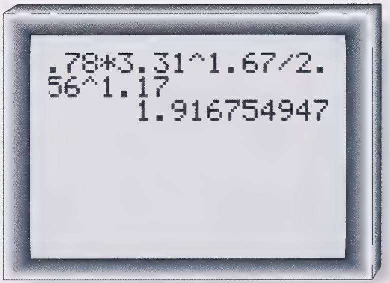
The hip length was 2.56 m.

Step 2: Calculate the speed.

$s = \dfrac{0.78\ell^{1.67}}{h^{1.17}}$

$= \dfrac{0.78 \times 3.31^{1.67}}{2.56^{1.17}}$

$\doteq 1.9$



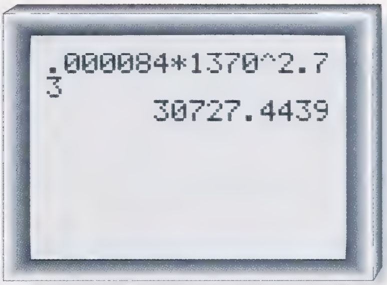
```
.78*3.31^1.67/2.
56^1.17
         1.916754947
```

The speed was about 1.9 m/s.

b. Textbook questions 1 and 2 of Investigation 2, "Calculating Masses," p. 45

1. $$\begin{aligned} M &= 0.000\,084\left(C_h + C_f\right)^{2.73} \\ &= 0.000\,084(650+720)^{2.73} \\ &= 0.000\,084 \times 1370^{2.73} \\ &\doteq 30\,727 \end{aligned}$$

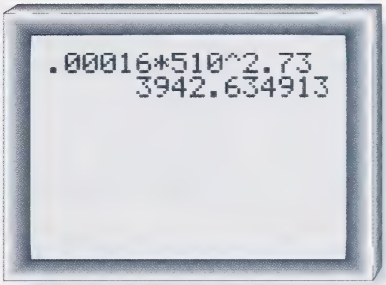
The Brachiosaurus had a mass of about 30 727 kg.



```
.000084*1370^2.7
3
          30727.4439
```

2. $$\begin{aligned} M &= 0.000\,16\left(C_f\right)^{2.73} \\ &= 0.000\,16 \times 510^{2.73} \\ &\doteq 3943 \end{aligned}$$

The Anatosaurus had a mass of about 3943 kg.



```
.00016*510^2.73
         3942.634913
```

15. a. Textbook question 1 of Investigation 1, "Comparing Distances," p. 43

1. **a.** $$\begin{aligned} 5\times\left(3\times10^5\right) &= 5\times3\times10^5 \\ &= 15\times10^5 \longleftarrow \text{This is not scientific notation.} \\ &= 1.5\times10\times10^5 \\ &= 1.5\times10^6 \end{aligned}$$

In 5 s, light travels 1.5×10^6 km.

b. $$\begin{aligned} 30\times\left(3\times10^5\right) &= 30\times3\times10^5 \\ &= 90\times10^5 \longleftarrow \text{This is not scientific notation.} \\ &= 9\times10\times10^5 \\ &= 9\times10^6 \end{aligned}$$

In 30 s, light travels 9.0×10^6 km.

Section 1: Activity 3 (continued)

c. $$\begin{aligned}300\times\left(3\times10^5\right)&=300\times3\times10^5\\&=900\times10^5\\&=9\times10^2\times10^5\\&=9\times10^7\end{aligned}$$

5 min = 300 s

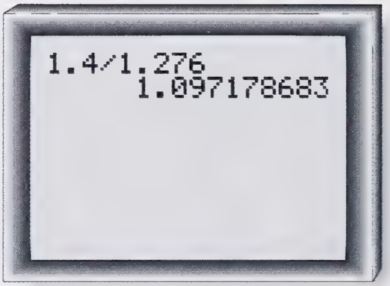
In 5 min, light travels 9×10^7 km.

b. Textbook question 1 of Investigation 2, “Comparing Sizes,” p. 43

1. $12\,760=1.276\times10^4$

$$\begin{aligned}\frac{1.4\times10^6}{1.276\times10^4}&=\frac{1.4}{1.276}\times\frac{10^6}{10^4}\\&\doteq1.1\times10^2\\&\doteq110\end{aligned}$$

About 110 Earths could fit along the diameter of the Sun.



```
1.4/1.276
          1.097178683
```

Section 1: Follow-up Activities

Extra Help

1. a. **Textbook questions 21 to 23 of “Review,” p. 46**

21. $$\begin{aligned}\sqrt{18}&=\sqrt{9\times2}\\&=\sqrt{3\times3\times2}\\&=\sqrt{3\times3}\times\sqrt{2}\\&=3\sqrt{2}\end{aligned}$$

22. $$\begin{aligned}\sqrt{32}&=\sqrt{4\times8}\\&=\sqrt{2\times2\times2\times2\times2}\\&=\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2}\\&=2\times2\times\sqrt{2}\\&=4\sqrt{2}\end{aligned}$$

23. $$\begin{aligned}\sqrt{500}&=\sqrt{2\times250}\\&=\sqrt{2\times25\times10}\\&=\sqrt{2\times5\times5\times2\times5}\\&=\sqrt{2\times2\times5\times5\times5}\\&=\sqrt{2\times2}\times\sqrt{5\times5}\times\sqrt{5}\\&=2\times5\times\sqrt{5}\\&=10\sqrt{5}\end{aligned}$$

b. Textbook questions 6 to 8 of "Chapter Check," p. 48

6.
$$\begin{aligned}\sqrt{50} &= \sqrt{5\times10}\\ &= \sqrt{5\times5\times2}\\ &= \sqrt{5\times5}\times\sqrt{2}\\ &= 5\sqrt{2}\end{aligned}$$

7.
$$\begin{aligned}\sqrt{44} &= \sqrt{4\times11}\\ &= \sqrt{2\times2\times11}\\ &= \sqrt{2\times2}\times\sqrt{11}\\ &= 2\sqrt{11}\end{aligned}$$

8.
$$\begin{aligned}\sqrt{80} &= \sqrt{8\times10}\\ &= \sqrt{2\times2\times2\times2\times5}\\ &= \sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{5}\\ &= 2\times2\times\sqrt{5}\\ &= 4\sqrt{5}\end{aligned}$$

2. a.
$$\begin{aligned}\sqrt[3]{54} &= \sqrt[3]{6\times9}\\ &= \sqrt[3]{2\times3\times3\times3}\\ &= \sqrt[3]{3\times3\times3}\times\sqrt[3]{2}\\ &= 3\sqrt[3]{2}\end{aligned}$$

b.
$$\begin{aligned}\sqrt[3]{16} &= \sqrt[3]{4\times4}\\ &= \sqrt[3]{2\times2\times2\times2}\\ &= \sqrt[3]{2\times2\times2}\times\sqrt[3]{2}\\ &= 2\sqrt[3]{2}\end{aligned}$$

c.
$$\begin{aligned}\sqrt[3]{24} &= \sqrt[3]{4\times6}\\ &= \sqrt[3]{2\times2\times2\times3}\\ &= \sqrt[3]{2\times2\times2}\times\sqrt[3]{3}\\ &= 2\sqrt[3]{3}\end{aligned}$$

d.
$$\begin{aligned}\sqrt[3]{81} &= \sqrt[3]{9\times9}\\ &= \sqrt[3]{3\times3\times3\times3}\\ &= \sqrt[3]{3\times3\times3}\times\sqrt[3]{3}\\ &= 3\sqrt[3]{3}\end{aligned}$$

3. a. $\left(\sqrt[3]{27}\right)^2 = 27^{\frac{2}{3}}$ **b.** $\left(\sqrt{36}\right)^3 = 36^{\frac{3}{2}}$ **c.** $\left(\sqrt[4]{625}\right)^3 = 625^{\frac{3}{4}}$

d. $\sqrt[3]{2^4} = 2^{\frac{4}{3}}$ **e.** $\sqrt{5^3} = 5^{\frac{3}{2}}$ **f.** $\sqrt[5]{(-5)^4} = (-5)^{\frac{4}{5}}$

Enrichment

1. a. Textbook questions 1 and 2 of Investigation 1, "The 45°–45°–90° Triangle," p. 26

1. a.
$$\begin{aligned}2^2+2^2 &= h^2\\ 4+4 &= h^2\\ 8 &= h^2\\ h &= \sqrt{8}\\ &= \sqrt{4\times2}\\ &= 2\sqrt{2}\end{aligned}$$

b.
$$\begin{aligned}3^2+3^2 &= h^2\\ 9+9 &= h^2\\ 18 &= h^2\\ h &= \sqrt{18}\\ &= \sqrt{9\times2}\\ &= 3\sqrt{2}\end{aligned}$$

c.
$$\begin{aligned}5^2+5^2 &= h^2\\ 25+25 &= h^2\\ 50 &= h^2\\ h &= \sqrt{50}\\ &= \sqrt{25\times2}\\ &= 5\sqrt{2}\end{aligned}$$

Enrichment (continued)

2. The length of the hypotenuse of a $45°–45°–90°$ triangle is $\sqrt{2}$ times the length of each leg.

b. Textbook questions 3 to 5 of Investigation 1, "The $45°–45°–90°$ Triangle," p. 26

3. **a.** $k = 7$
$h = 7\sqrt{2}$

b. $k = 10$
$h = 10\sqrt{2}$

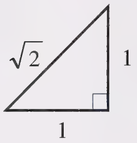
c. $a = 6$
$b = 6$

d.

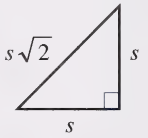
$$\begin{aligned} 8 &= \sqrt{64} \\ &= \sqrt{2} \times \sqrt{32} \end{aligned}$$

$$\begin{aligned} \therefore\ c = d &= \sqrt{32} \\ &= \sqrt{16 \times 2} \\ &= 4\sqrt{2} \end{aligned}$$

4. If the length of each leg of a $45°–45°–90°$ triangle is 1, the hypotenuse is $\sqrt{2}$.

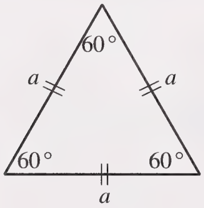


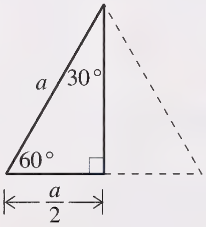
5. If the length of each leg of a $45°–45°–90°$ triangle is s, the hypotenuse is $s\sqrt{2}$.



2. **a.** **Textbook question 2 of Investigation 2, "The $30°–60°–90°$ Triangle," p. 26**

2.





In a $30°–60°–90°$ triangle, the hypotenuse is twice the length of the side opposite the 30° angle. Or, in a $30°–60°–90°$ triangle, the side opposite the 30° angle is half the length of the hypotenuse.

b. Textbook questions 1 and 3 of Investigation 2, "The $30°–60°–90°$ Triangle," pp. 26 and 27

1. a.

$$\begin{aligned} a &= \frac{4}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} 2^2 + b^2 &= 4^2 \\ 4 + b^2 &= 16 \\ b^2 &= 12 \\ b &= \sqrt{12} \\ &= \sqrt{4 \times 3} \\ &= 2\sqrt{3} \end{aligned}$$

b.

$$\begin{aligned} y &= \frac{10}{2} \\ &= 5 \end{aligned}$$

$$\begin{aligned} x^2 + 5^2 &= 10^2 \\ x^2 + 25 &= 100 \\ x^2 &= 75 \\ x &= \sqrt{75} \\ &= \sqrt{25 \times 3} \\ &= 5\sqrt{3} \end{aligned}$$

c.

$$\begin{aligned} e &= \frac{8}{2} \\ &= 4 \end{aligned}$$

$$\begin{aligned} d^2 + 4^2 &= 8^2 \\ d^2 + 16 &= 64 \\ d^2 &= 48 \\ d &= \sqrt{16 \times 3} \\ &= 4\sqrt{3} \end{aligned}$$

d.

$$\begin{aligned} g &= \frac{2}{2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} 1^2 + k^2 &= 2^2 \\ 1 + k^2 &= 4 \\ k^2 &= 3 \\ k &= \sqrt{3} \end{aligned}$$

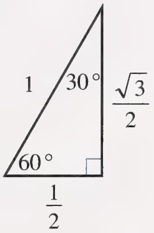
3. In a $30°–60°–90°$ triangle, the length of the side opposite the 60° angle is $\sqrt{3}$ times the length of the side opposite the 30° angle.

c. Textbook questions 4 to 6 of Investigation 2, "The $30°–60°–90°$ Triangle," p. 27

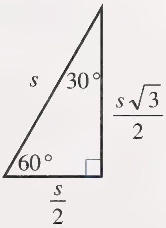
4. a. $a = 6\sqrt{3}$ and $h = 12$ **b.** $k = 8\sqrt{3}$ and $h = 16$

c. $a = 3$ and $h = 6$

5. The length of the side opposite the 30° angle is $\frac{1}{2}$. The length of the side opposite the 60° angle is $\frac{\sqrt{3}}{2}$.



6. The length of the side opposite the 30° angle is $\frac{s}{2}$. The length of the side opposite the 60° angle is $\frac{s\sqrt{3}}{2}$.



Section 2: Activity 1

1. a. **Textbook questions 1 and 2 of Investigation 1, "Patterns in Numbers and Letters," p. 52**

1. a. The next term is 3 more than the previous term.

4, 7, 10, 13, 16, 19, 22

+3 +3 +3

b. The next term is 2 times the previous term.

3, 6, 12, 24, 48, 96, 192

×2 ×2 ×2

c. The next term is equal to the previous term plus the term number of the term.

1, 2, 4, 7, 11, 16, 22, 29, 37

+6 +7 +8

6th term 7th term 8th term

d. The next terms are alternately 3 times the previous term and 0.5 times the previous term.

4, 12, 6, 18, 9, 27, 13.5, 40.5

×3 ×0.5 ×3 ×0.5 ×3 ×0.5 ×3

e. The next terms are alternately 1 more than the previous term and 3 times the previous term.

2, 3, 9, 10, 30, 31, 93, 94

+1 ×3 +1 ×3 +1 ×3 +1

f. The first two terms are 3 and 4. Each successive term is the sum of the previous two terms.

3, 4, 7, 11, 18, 29, 47, 76, 123

18 + 29 29 + 47 47 + 76

g. The next term is 18 more than the previous term.

9, 27, 45, 63, 81 , 99 , 117

+18 +18 +18

2. **a.** The first term is A. Each successive term is found by going forward in the alphabet 3 letters.

A, D, G, J, M , P , S

b. The first term is A. Each successive term is found by alternately going forward in the alphabet 3 letters, then going backward in the alphabet 2 letters.

A, D, B, E, C, F, D, G , E , H

c. The first term is A. Each successive term is found by alternately going forward in the alphabet 2 letters, then going forward 1 letter.

A, C, D, F, G, I, J, L , M , O

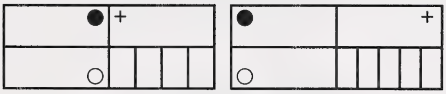
d. The terms are alternately numbers and letters. The first number is 1. The difference between successive numbers is 3, 4, 5, …. The first letter is Z. Successive letters are found by going backward in the alphabet 1 letter.

1, Z, 4, Y, 8, X, 13, W , 19 , V

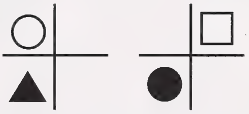
+3 +4 +5 +6

b. Textbook questions 2 and 3 of Investigation 2, "Patterns in Diagrams," p. 52

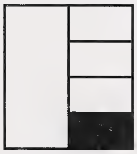
2. **a.**



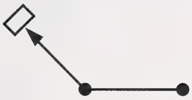
b.



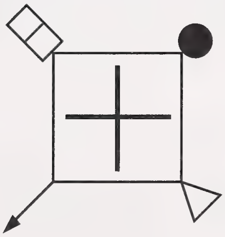
3. **a.**



b.



c.



Section 2: Activity 1 (continued)

2. **Textbook questions 1 to 3 of Investigation 4, "Generating Sequences," p. 63**

   1. 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, $\underbrace{4, 2, 1,}\ \underbrace{4, 2, 1,}$ …

      The repeating pattern of three numbers is 4, 2, and 1.

   2. 25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, $\underbrace{4, 2, 1,}\ \underbrace{4, 2, 1}$

      The repeating pattern of three numbers is 4, 2, and 1.

   3. The sequences will vary depending on the number chosen, but you will always get the repeating pattern 4, 2, 1.

3. **Textbook questions 1 to 7 of "Inquire," p. 65**

   1. Each term is equal to the number of the term.

   2. $t_5 = 5$, $t_6 = 6$, and $t_7 = 7$

   3. Each term is twice the preceding term.

   4. $t_5 = 16$, $t_6 = 32$, and $t_7 = 64$

   5. Two other examples of an Archimedial spiral include a roll of toilet paper and a jelly-roll dessert.

   6. Two other examples of an exponential spiral are a chambered nautilus shell and the central florets of a daisy.

   7. The terms in sequences such as 1, 2, 4, 8, … or 1, 3, 9, 27, … grow exponentially (at an increasing rate). The general term of sequences such as these have exponents greater than 1.

4. Textbook questions 7, 9, 11, and 13 of "Practice," p. 66

7. **Step 1:** Make a table relating each term to its term number. Then find a pattern in the growth of the terms.

Term Number (n)	1	2	3	4
Term (t_n)	5	10	15	20

+5 +5 +5

Step 2: Use the pattern you discovered in Step 1 and the guess and check strategy to write a formula for the general term.

$$t_1 = 5(\mathbf{1}) = 5$$
$$t_2 = 5(\mathbf{2}) = 10$$
$$t_3 = 5(\mathbf{3}) = 15$$
$$t_4 = 5(\mathbf{4}) = 20$$

The formula is $t_n = 5n$.

9. **Step 1:** Make a table relating each term to its term number. Then find a pattern in the growth of the terms.

Term Number (n)	1	2	3	4
Term (t_n)	6	5	4	3

−1 −1 −1

Step 2: Use the pattern you discovered in Step 1 and the guess and check strategy to write a formula for the general term.

$$t_1 = 7 - \mathbf{1} = 6$$
$$t_2 = 7 - \mathbf{2} = 5$$
$$t_3 = 7 - \mathbf{3} = 4$$
$$t_4 = 7 - \mathbf{4} = 3$$

The formula is $t_n = 7 - n$.

Section 2: Activity 1 (continued)

11. **Step 1:** Make a table relating each term to its term number. Then find a pattern in the growth of the terms.

Term Number (n)	1	2	3	4	5
Term (t_n)	2	4	6	8	10

+2 +2 +2 +2

Step 2: Use the pattern you discovered in Step 1 and the guess and check strategy to write a formula for the general term.

$$t_1 = 2(\mathbf{1}) = 2$$
$$t_2 = 2(\mathbf{2}) = 4$$
$$t_3 = 2(\mathbf{3}) = 6$$
$$t_4 = 2(\mathbf{4}) = 8$$
$$t_5 = 2(\mathbf{5}) = 10$$

The formula is $t_n = 2n$.

13. **Step 1:** Make a table relating each term to its term number. Then find a pattern in the growth of the terms.

Term Number (n)	1	2	3	4	5
Term (t_n)	−1	0	1	2	3

+1 +1 +1 +1

Step 2: Use the pattern you discovered in Step 1 and the guess and check strategy to write a formula for the general term.

$$t_1 = \mathbf{1} - 2 = -1$$
$$t_2 = \mathbf{2} - 2 = 0$$
$$t_3 = \mathbf{3} - 2 = 1$$
$$t_4 = \mathbf{4} - 2 = 2$$
$$t_5 = \mathbf{5} - 2 = 3$$

The formula is $t_n = n - 2$.

5. Textbook questions 28, 30, 32, and 34 of "Practice," p. 67

28.
$$\begin{aligned} t_n &= 2n+7 \\ t_6 &= 2(6)+7 \\ &= 12+7 \\ &= 19 \end{aligned} \qquad \begin{aligned} t_n &= 2n+7 \\ t_{15} &= 2(15)+7 \\ &= 30+7 \\ &= 37 \end{aligned}$$

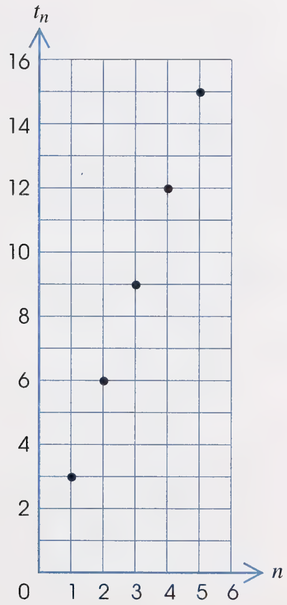
30.
$$\begin{aligned} t_n &= 12+5n \\ t_3 &= 12+5(3) \\ &= 12+15 \\ &= 27 \end{aligned} \qquad \begin{aligned} t_n &= 12+5n \\ t_{10} &= 12+5(10) \\ &= 12+50 \\ &= 62 \end{aligned}$$

32.
$$\begin{aligned} t_n &= n^2+4 \\ t_3 &= 3^2+4 \\ &= 9+4 \\ &= 13 \end{aligned} \qquad \begin{aligned} t_n &= n^2+4 \\ t_7 &= 7^2+4 \\ &= 49+4 \\ &= 53 \end{aligned}$$

34.
$$\begin{aligned} t_n &= (n-3)^2 \\ t_2 &= (2-3)^2 \\ &= (-1)^2 \\ &= 1 \end{aligned} \qquad \begin{aligned} t_n &= (n-3)^2 \\ t_{15} &= (15-3)^2 \\ &= 12^2 \\ &= 144 \end{aligned}$$

6. Textbook questions 1, 3, and 5 of "Practice," p. 66

1.
$$\begin{aligned} t_n &= 3n \\ t_1 &= 3\times 1 \\ &= 3 \end{aligned} \qquad \begin{aligned} t_n &= 3n \\ t_2 &= 3\times 2 \\ &= 6 \end{aligned} \qquad \begin{aligned} t_n &= 3n \\ t_3 &= 3\times 3 \\ &= 9 \end{aligned} \qquad \begin{aligned} t_n &= 3n \\ t_4 &= 3\times 4 \\ &= 12 \end{aligned} \qquad \begin{aligned} t_n &= 3n \\ t_5 &= 3\times 5 \\ &= 15 \end{aligned}$$



Section 2: Activity 1 (continued)

3.

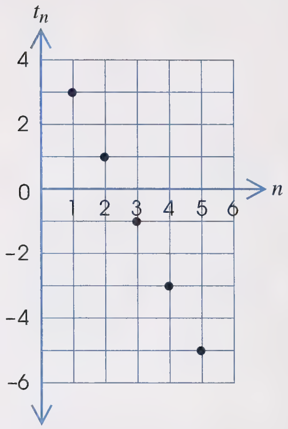
$$\begin{aligned} t_n &= 5-2n \\ t_1 &= 5-2(1) \\ &= 5-2 \\ &= 3 \end{aligned}$$

$$\begin{aligned} t_n &= 5-2n \\ t_2 &= 5-2(2) \\ &= 5-4 \\ &= 1 \end{aligned}$$

$$\begin{aligned} t_n &= 5-2n \\ t_3 &= 5-2(3) \\ &= 5-6 \\ &= -1 \end{aligned}$$

$$\begin{aligned} t_n &= 5-2n \\ t_4 &= 5-2(4) \\ &= 5-8 \\ &= -3 \end{aligned}$$

$$\begin{aligned} t_n &= 5-2n \\ t_5 &= 5-2(5) \\ &= 5-10 \\ &= -5 \end{aligned}$$



5.

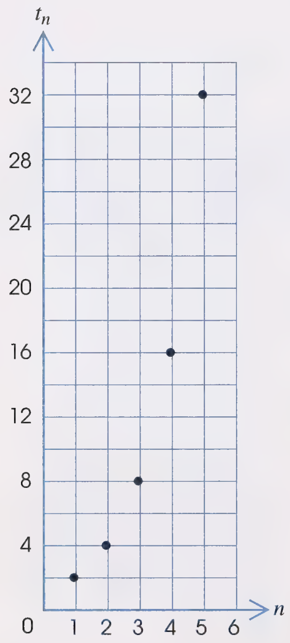
$$\begin{aligned} t_n &= 2^n \\ t_1 &= 2^1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} t_n &= 2^n \\ t_2 &= 2^2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} t_n &= 2^n \\ t_3 &= 2^3 \\ &= 8 \end{aligned}$$

$$\begin{aligned} t_n &= 2^n \\ t_4 &= 2^4 \\ &= 16 \end{aligned}$$

$$\begin{aligned} t_n &= 2^n \\ t_5 &= 2^5 \\ &= 32 \end{aligned}$$



7. Textbook questions 1, 3, and 5 of "Practice," p. 66

1. **Step 1:** Put the calculator in sequence (seq) function mode.

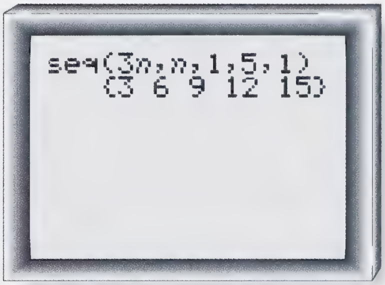
   Press MODE; use the arrow keys to select "Seq" on the fourth line; and press ENTER.

   Step 2: List the terms of the sequence.

   Press 2nd [LIST], select the OPS menu, and press 5 to select "seq(."

   Now, press the following sequence of keys to state the general term, the variable, the first term number, the ending term number, and the desired increments:

   3 X,T,Θ,*n* , X,T,Θ,*n* , 1 ,
   5 , 1) ENTER



```
seq(3n,n,1,5,1)
     {3 6 9 12 15}
```

   The first five terms of the sequence are 3, 6, 9, 12, and 15.

   Step 3: Display the default settings by pressing WINDOW. Use the delete key, DEL, and the insert key, 2nd [INS], to make the following edits to the default settings:

   - All the term numbers (n) are natural numbers and you want to show the first five terms of the sequence. Therefore, use the settings $n\text{Min} = 1$ and $n\text{Max} = 5$.
   - All the term numbers (n) will be plotted on the x-axis. Therefore, use the settings $\text{Xmin} = 0$ and $\text{Xmax} = 5$.
   - You know that the first five terms of the sequence are 3, 6, 9, 12, and 15, and you know that the terms will be plotted on the y-axis. Therefore, use the settings $\text{Ymin} = 0$ and $\text{Ymax} = 15$.

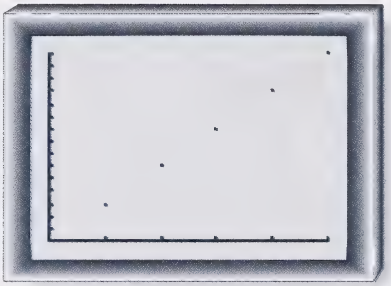
Section 2: Activity 1 (continued)

Step 4: Display the sequence Y = editor by pressing Y=. The TI-83 calculator has three sequence functions: u(n), v(n), and w(n). You will use only u(n).

Move the cursor to the right of "u(n)="; then press 3 X,T,Θ,n. Ensure that the icon to the left of u(n) is three discrete dots.

Step 5: Graph t_n versus n.

Press GRAPH. The following graph should appear:

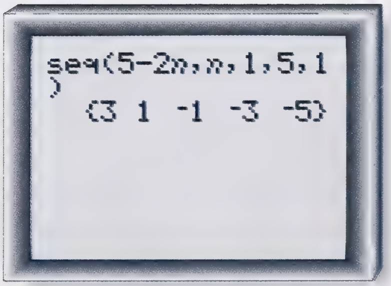


3. **Step 1:** Ensure that the calculator is in sequence (Seq) function mode.

Step 2: Press 2nd [LIST]; select the OPS menu, and press 5 to select "seq(."

Now, press the following key sequence:

5 − 2 X,T,Θ,n , X,T,Θ,n ,
1 , 5 , 1) ENTER



```
seq(5-2n,n,1,5,1
)
  {3 1 -1 -3 -5}
```

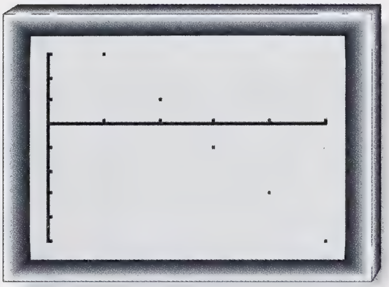
The first five terms of the sequence are 3, 1, −1, −3, and −5.

Step 3: Press WINDOW and edit the settings so that nMin $= 1$, nMax $= 5$, Xmin $= 0$, Xmax $= 5$, Ymin $= -5$, and Ymax $= 3$.

Step 4: Display the sequence Y = editor by pressing Y= .

Move the cursor to the right of "u (n) =." Press 5 − 2 X,T,Θ,n. Ensure that the dots to the left of u(n) are three discrete dots.

Step 5: Graph t_n versus n by pressing GRAPH. The following graph should appear:

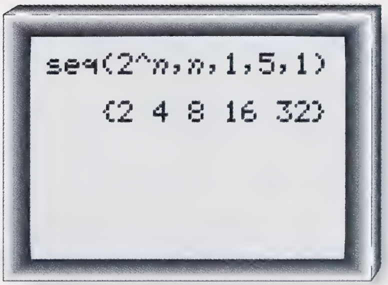


5. **Step 1:** Ensure that the calculator is in sequence (Seq) function mode.

Step 2: Press 2nd [LIST], select the OPS menu, and press 5 to select "seq(."

Now, press the following key sequence:

2 ^ X,T,Θ,n , X,T,Θ,n , 1 ,
5 , 1) ENTER



```
seq(2^n,n,1,5,1)
   {2 4 8 16 32}
```

The first five terms of the sequence are 2, 4, 8, 16, and 32.

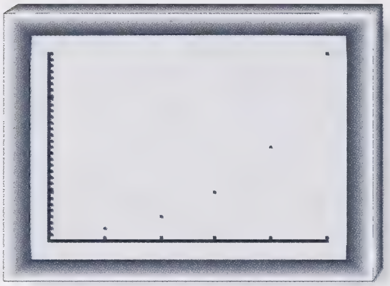
Section 2: Activity 1 (continued)

Step 3: Press WINDOW and edit the settings so that nMin $= 1$, nMax $= 5$, Xmin $= 0$, Xmax $= 5$, Ymin $= 0$, and Ymax $= 32$.

Step 4: Display the sequence Y = editor by pressing Y=.

Move the cursor to the right of "u$(n)=$." Press 2 ^ X,T,Θ,n. Ensure the dots to the left of u(n) are three discrete dots.

Step 5: Graph t_n versus n by pressing GRAPH. The following graph should appear:



8. Textbook questions 35 to 37 of "Practice," p. 67

35. $t_1 = 4$

$$\begin{aligned} t_n &= t_{n-1} + 3 \\ t_2 &= t_{2-1} + 3 \\ &= t_1 + 3 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

$$\begin{aligned} t_n &= t_{n-1} + 3 \\ t_3 &= t_{3-1} + 3 \\ &= t_2 + 3 \\ &= 7 + 3 \\ &= 10 \end{aligned}$$

$$\begin{aligned} t_n &= t_{n-1} + 3 \\ t_4 &= t_{4-1} + 3 \\ &= t_3 + 3 \\ &= 10 + 3 \\ &= 13 \end{aligned}$$

$$\begin{aligned} t_n &= t_{n-1} + 3 \\ t_5 &= t_{5-1} + 3 \\ &= t_4 + 3 \\ &= 13 + 3 \\ &= 16 \end{aligned}$$

The first five terms are 4, 7, 10, 13, and 16.

36. $t_1 = -1$

$$\begin{aligned} t_n &= 2t_{n-1} + 5 \\ t_2 &= 2t_{2-1} + 5 \\ &= 2t_1 + 5 \\ &= 2(-1) + 5 \\ &= -2 + 5 \\ &= 3 \end{aligned}$$

$$\begin{aligned} t_n &= 2t_{n-1} + 5 \\ t_3 &= 2t_{3-1} + 5 \\ &= 2t_2 + 5 \\ &= 2(3) + 5 \\ &= 6 + 5 \\ &= 11 \end{aligned}$$

$$\begin{aligned} t_n &= 2t_{n-1} + 5 \\ t_4 &= 2t_{4-1} + 5 \\ &= 2t_3 + 5 \\ &= 2(11) + 5 \\ &= 22 + 5 \\ &= 27 \end{aligned}$$

$$\begin{aligned} t_n &= 2t_{n-1} + 5 \\ t_5 &= 2t_{5-1} + 5 \\ &= 2t_4 + 5 \\ &= 2(27) + 5 \\ &= 54 + 5 \\ &= 59 \end{aligned}$$

The first five terms are −1, 3, 11, 27, and 59.

37. $t_1 = 3$

$$\begin{aligned} t_n &= t_{n-1} - 2 \\ t_2 &= t_{2-1} - 2 \\ &= t_1 - 2 \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} t_n &= t_{n-1} - 2 \\ t_3 &= t_{3-1} - 2 \\ &= t_2 - 2 \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

$$\begin{aligned} t_n &= t_{n-1} - 2 \\ t_4 &= t_{4-1} - 2 \\ &= t_3 - 2 \\ &= -1 - 2 \\ &= -3 \end{aligned}$$

$$\begin{aligned} t_n &= t_{n-1} - 2 \\ t_5 &= t_{5-1} - 2 \\ &= t_4 - 2 \\ &= -3 - 2 \\ &= -5 \end{aligned}$$

The first five terms are 3, 1, −1, −3, and -5.

9. Textbook questions 46 and 47 of "Applications and Problem Solving," p. 67

46. a.

$$\begin{array}{r} 165.0 \\ +\quad 4.5 \\ \hline 169.5 \end{array} \qquad \begin{array}{r} 169.5 \\ +\quad 4.5 \\ \hline 174.0 \end{array}$$

The total production was 169.5 t at the end of 1993. The total production was 174.0 t at the end of 1994.

Section 2: Activity 1 (continued)

b. First, find the general term.

$$\begin{aligned} t_1 &= 165 \\ t_2 &= 165 + 4.5 \times 1 \\ t_3 &= 165 + 4.5 \times 2 \\ \therefore\ t_n &= 165 + 4.5(n-1) \end{aligned}$$

Find what the total production should be by the end of 2010.

$$\begin{aligned} t_n &= 165 + 4.5(n-1) \\ t_{19} &= 165 + 4.5(19-1) \\ &= 165 + 4.5(18) \\ &= 165 + 81 \\ &= 246 \end{aligned}$$

There are 19 years of production between 1992 and 2010.

The total production should be 246 t by the end of 2010.

Find what the total production should be by the end of 2020.

$$\begin{aligned} t_n &= 165 + 4.5(n-1) \\ t_{29} &= 165 + 4.5(29-1) \\ &= 165 + 4.5(28) \\ &= 165 + 126 \\ &= 291 \end{aligned}$$

There are 29 years of production between 1992 and 2020.

The total production should be 291 t by the end of 2020.

47. a. $10 + 0.25 = 10.25$ ← after 1 week

$10.25 + 0.25 = 10.5$ ← after 2 weeks

$10.5 + 0.25 = 10.75$ ← after 3 weeks

$10.75 + 0.25 = 11$ ← after 4 weeks

$11 + 0.25 = 11.25$ ← after 5 weeks

$11.25 + 0.25 = 11.5$ ← after 6 weeks

Tania's hair will be 10.25 cm long in 1 week, 10.5 cm long in 2 weeks, and 11.5 cm long in 6 weeks.

b. $t_1 = 10$

$t_2 = 10 + 0.25 \times 1$

$t_3 = 10 + 0.25 \times 2$

$\therefore\ t_n = 10 + 0.25(n-1)$

Notice that the number of weeks is 1 less than the term number.

$$\begin{aligned} 10 + 0.25(n-1) &= 15 \\ 0.25(n-1) &= 5 \\ \frac{0.25(n-1)}{0.25} &= \frac{5}{0.25} \\ n-1 &= 20 \\ n &= 21 \end{aligned}$$

Recall that the number of weeks is 1 less than the term number. Therefore, it will take 20 weeks for Tania's hair to grow 5 cm.

10. a. Textbook question "Explore: Complete the Table," p. 72

	Term				
	t_1	t_2	t_3	t_4	t_5
Sequence	5	15	25	35	45
Sequence Expressed Using 5 and 10	5	$5+1(10)$	$5+2(10)$	$5+3(10)$	$5+4(10)$
Sequence Expressed Using *a* and *d*	a	$a+1d$	$a+2d$	$a+3d$	$a+4d$

b. Textbook questions 1 to 4 of "Inquire," p. 72

1. $a = 5$ and $d = 10$

2. $t_5 = a + 4d$ $\quad t_8 = a + 7d$ $\quad t_9 = a + 8d$

3.

$$\begin{aligned} t_8 &= a + 7d \\ &= 5 + 7(10) \\ &= 5 + 70 \\ &= 75 \end{aligned} \qquad \begin{aligned} t_9 &= a + 8d \\ &= 5 + 8(10) \\ &= 5 + 80 \\ &= 85 \end{aligned}$$

4. The speed at which the object falls becomes constant and does not increase.

Section 2: Activity 1 (continued)

11. a. Textbook questions 1, 3, and 5 of "Practice," p. 74

1.

$$\begin{aligned} t_n &= a+(n-1)d \\ t_4 &= 3+(4-1)4 \\ &= 3+3\times 4 \\ &= 3+12 \\ &= 15 \end{aligned}$$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_5 &= 3+(5-1)4 \\ &= 3+4\times 4 \\ &= 3+16 \\ &= 19 \end{aligned}$$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_6 &= 3+(6-1)4 \\ &= 3+5\times 4 \\ &= 3+20 \\ &= 23 \end{aligned}$$

The next three terms are 15, 19, and 23.

3.

$$\begin{aligned} t_n &= a+(n-1)d \\ t_4 &= -23+(4-1)5 \\ &= -23+3\times 5 \\ &= -23+15 \\ &= -8 \end{aligned}$$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_5 &= -23+(5-1)5 \\ &= -23+4\times 5 \\ &= -23+20 \\ &= -3 \end{aligned}$$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_6 &= -23+(6-1)5 \\ &= -23+5\times 5 \\ &= -23+25 \\ &= 2 \end{aligned}$$

The next three terms are -8, -3, and $+2$.

5.

$$\begin{aligned} t_n &= a+(n-1)d \\ t_4 &= 5.8+(4-1)1.4 \\ &= 5.8+3\times 1.4 \\ &= 5.8+4.2 \\ &= 10 \end{aligned}$$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_5 &= 5.8+(5-1)1.4 \\ &= 5.8+4\times 1.4 \\ &= 5.8+5.6 \\ &= 11.4 \end{aligned}$$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_6 &= 5.8+(6-1)1.4 \\ &= 5.8+5\times 1.4 \\ &= 5.8+7 \\ &= 12.8 \end{aligned}$$

The next three terms are 10, 11.4, and 12.8.

b. Textbook questions 7 to 11 of "Practice," p. 74

7. This sequence is arithmetic, where $a = 5$ and $d = 4$.

8. This sequence is **not** arithmetic.

9. This sequence is **not** arithmetic.

10. This sequence is arithmetic, where $a = -1$ and $d = -3$.

11. This sequence is **not** arithmetic.

c. Textbook questions 15 to 18 of "Practice," p. 75

15. If $a = 7$ and $d = 2$, then the first five terms of the sequence are 7, 9, 11, 13, and 15.

16. If $a = 3$ and $d = 4$, then the first five terms of the sequence are 3, 7, 11, 15, and 19.

17. If $a = -4$ and $d = 6$, then the first five terms of the sequence are -4, 2, 8, 14, and 20.

18. If $a = 2$ and $d = -3$, then the first five terms of the sequence are 2, -1, -4, -7, and -10.

d. Textbook questions 23 to 27 of "Practice," p. 75

23. $a = 6$ and $d = 2$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{10} &= 6 + (10-1)2 \\ &= 6 + 9 \times 2 \\ &= 6 + 18 \\ &= 24 \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{34} &= 6 + (34-1)2 \\ &= 6 + 33 \times 2 \\ &= 6 + 66 \\ &= 72 \end{aligned}$$

24. $a = 12$ and $d = 4$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{18} &= 12 + (18-1)4 \\ &= 12 + 17 \times 4 \\ &= 12 + 68 \\ &= 80 \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{41} &= 12 + (41-1)4 \\ &= 12 + 40 \times 4 \\ &= 12 + 160 \\ &= 172 \end{aligned}$$

25. $a = 9$ and $d = 7$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{9} &= 9 + (9-1)7 \\ &= 9 + 8 \times 7 \\ &= 9 + 56 \\ &= 65 \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{100} &= 9 + (100-1)7 \\ &= 9 + 99 \times 7 \\ &= 9 + 693 \\ &= 702 \end{aligned}$$

Section 2: Activity 1 (continued)

26. $a = -10$ and $d = 3$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{11} &= -10 + (11-1)3 \\ &= -10 + 10 \times 3 \\ &= -10 + 30 \\ &= 20 \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{22} &= -10 + (22-1)3 \\ &= -10 + 21 \times 3 \\ &= -10 + 63 \\ &= 53 \end{aligned}$$

27. $a = -4$ and $d = -5$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{18} &= -4 + (18-1)(-5) \\ &= -4 + 17(-5) \\ &= -4 + (-85) \\ &= -89 \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{66} &= -4 + (66-1)(-5) \\ &= -4 + 65(-5) \\ &= -4 + (-325) \\ &= -329 \end{aligned}$$

12. Textbook questions 28 to 30 of "Practice," p. 75

28. Step 1: Find the general term.

$a = 5$ and $d = -6$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 5 + (n-1)(-6) \end{aligned}$$

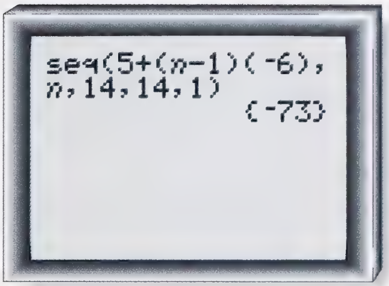
Note: *It is not necessary to simplify the general term further.*

Step 2: Ensure the calculator is in the sequence (Seq) function mode.

Step 3: Press 2nd [LIST], select the OPS menu, and press 5 to select “seq(.”

Then press the following key sequence:

5 + (X,T,Θ,*n* − 1)
((−) 6) , X,T,Θ,*n* ,
1 4 , 1 4 , 1)
ENTER



```
seq(5+(n-1)(-6),
n,14,14,1)
                {-73}
```

The 14th term is -73.

29. **Step 1:** Find the general term.

$a = 7$ and $d = 3$

$$\begin{aligned} t_n &= a+(n-1)d \\ &= 7+(n-1)3 \end{aligned}$$

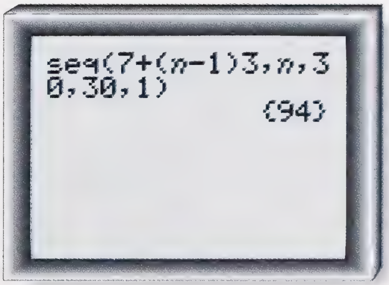
Note: *It is not necessary to simplify the general term further.*

Step 2: Ensure the calculator is in the sequence (Seq) function mode.

Step 3: Press 2nd [LIST], select the OPS menu, and press 5 to select “seq(.”

Then press the following key sequence:

7 + (X,T,Θ,*n* − 1)
3 , X,T,Θ,*n* , 3 0 ,
3 0 , 1) ENTER



```
seq(7+(n-1)3,n,3
0,30,1)
                 {94}
```

The 30th term is 94.

Section 2: Activity 1 (continued)

30. **Step 1:** Find the general term.

$a = 10$ and $d = -2$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 10 + (n-1)(-2) \end{aligned}$$

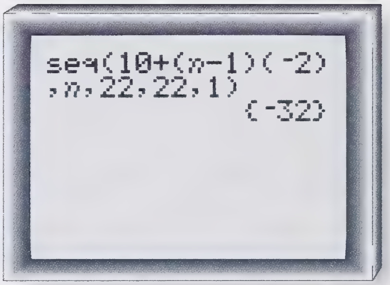
***Note:** It is not necessary to simplify the general term further.*

Step 2: Ensure the calculator is in the sequence (Seq) function mode.

Step 3: Press 2nd [LIST], select the OPS menu, and press 5 to select "seq(."

Then press the following key sequence:

1 0 + (X,T,Θ,*n* − 1)
((−) 2) , X,T,Θ,*n* ,
2 2 , 2 2 , 1)
ENTER



```
seq(10+(n−1)(-2)
,n,22,22,1)
              {-32}
```

The 22nd term is −32.

13. a. Textbook questions 32 to 35 of "Practice," p. 75

32. $a = 10$, $d = 5$, and $t_n = 250$

$$\begin{aligned} t_n &= a + (n-1)d \\ 250 &= 10 + (n-1)5 \\ 250 &= 10 + 5n - 5 \\ 250 &= 5 + 5n \\ 245 &= 5n \\ 49 &= n \end{aligned}$$

There are 49 terms in the sequence.

33. $a = 1$, $d = 3$, and $t_n = 121$

$$\begin{aligned} t_n &= a + (n-1)d \\ 121 &= 1 + (n-1)3 \\ 121 &= 1 + 3n - 3 \\ 121 &= 3n - 2 \\ 123 &= 3n \\ 41 &= n \end{aligned}$$

There are 41 terms in the sequence.

34. $a=40,\ d=-2,\text{ and } t_n=-30$

$$\begin{aligned}t_n &= a+(n-1)d\\ -30 &= 40+(n-1)(-2)\\ -30 &= 40-2n+2\\ -30 &= 42-2n\\ 2n-30 &= 42\\ 2n &= 72\\ n &= 36\end{aligned}$$

There are 36 terms in the sequence.

35. $a=-11,\ d=4,\text{ and } t_n=153$

$$\begin{aligned}t_n &= a+(n-1)d\\ 153 &= -11+(n-1)4\\ 153 &= -11+4n-4\\ 153 &= 4n-15\\ 168 &= 4n\\ 42 &= n\end{aligned}$$

There are 42 terms in the sequence.

b. Textbook questions 49, 50, 56, and 64 of "Applications and Problem Solving," pp. 75 and 76

49. $a=15,\ d=5,\text{ and } t_n=450$

$$\begin{aligned}t_n &= a+(n-1)d\\ 450 &= 15+(n-1)5\\ 450 &= 15+5n-5\\ 450 &= 10+5n\\ 440 &= 5n\\ 88 &= n\end{aligned}$$

There are 88 multiples of 5 between 15 and 450.

50. $d=15,\ n=18,\text{ and } t_{18}=262$

$$\begin{aligned}t_n &= a+(n-1)d\\ t_{18} &= a+(18-1)15\\ 262 &= a+17\times 15\\ 262 &= a+255\\ 7 &= a\end{aligned}$$

The first term is 7.

56. a. $a=8$ and $d=2$

b. $$\begin{aligned}t_n &= a+(n-1)d\\ &= 8+(n-1)2\\ &= 8+2n-2\\ &= 2n+6\end{aligned}$$

Section 2: Activity 1 (continued)

c.

$$\begin{aligned} t_n &= 2n+6 \\ t_{22} &= 2(22)+6 \\ &= 44+6 \\ &= 50 \end{aligned}$$

On the 22nd day of practice, the team ran 50 windsprints—a total distance of $50 \times 40\ \text{m} = 2000\ \text{m}$.

64. $a = 4$ and $d = 9$

$$\begin{aligned} t_n &= a+(n-1)d \\ &= 4+(n-1)9 \\ &= 4+9n-9 \\ &= 9n-5 \end{aligned}$$

The five numbers given are consecutive. Because the common difference is 9, only one of the given numbers can be a term of the sequence.

In a sequence, n must be a natural number. Therefore, when the number that is a term of the sequence is substituted into the general term, it results in n being a natural number.

Is 316 a term?

$$\begin{aligned} t_n &= 9n-5 \\ 316 &= 9n-5 \\ 321 &= 9n \\ 35.\dot{6} &= n \longleftarrow \text{not a natural number} \end{aligned}$$

No, 316 is not a term.

Is 317 a term?

$$\begin{aligned} t_n &= 9n-5 \\ 317 &= 9n-5 \\ 322 &= 9n \\ 35.\dot{7} &= n \longleftarrow \text{not a natural number} \end{aligned}$$

No, 317 is not a term.

Is 318 a term?

$$\begin{aligned} t_n &= 9n-5 \\ 318 &= 9n-5 \\ 323 &= 9n \\ 35.\dot{8} &= n \longleftarrow \text{not a natural number} \end{aligned}$$

No, 318 is not a term.

Is 319 a term?

$$\begin{aligned} t_n &= 9n-5 \\ 319 &= 9n-5 \\ 324 &= 9n \\ 36 &= n \longleftarrow \text{a natural number} \end{aligned}$$

Yes, 319 is a term.

14. Textbook questions 38 to 42 of "Practice," p. 75

38. $a=1$, $n=3$, and $t_3=25$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_3 &= 1+(3-1)d \\ 25 &= 1+2d \\ 24 &= 2d \\ 12 &= d \end{aligned}$$

The sequence is 1, 13, 25.

39. $a=14$, $n=4$, and $t_4=32$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_4 &= 14+(4-1)d \\ 32 &= 14+3d \\ 18 &= 3d \\ 6 &= d \end{aligned}$$

The sequence is 14, 20, 26, 32.

40. $a=-3$, $n=4$, and $t_4=-60$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_4 &= -3+(4-1)d \\ -60 &= -3+3d \\ -57 &= 3d \\ -19 &= d \end{aligned}$$

The sequence is -3, -22, -41, -60.

41. $a=-1.5$, $n=5$, and $t_5=4.5$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_5 &= -1.5+(5-1)d \\ 4.5 &= -1.5+4d \\ 6 &= 4d \\ 1.5 &= d \end{aligned}$$

The sequence is -1.5, 0, 1.5, 3, 4.5.

42. $a=2$, $n=6$, and $t_6=107$

$$\begin{aligned} t_n &= a+(n-1)d \\ t_6 &= 2+(6-1)d \\ 107 &= 2+5d \\ 105 &= 5d \\ 21 &= d \end{aligned}$$

The sequence is 2, 23, 44, 65, 86, 107.

15. a. Textbook questions 1, 3, 5, 7, 9, 11, and 13 of Investigation 1, "Writing Terms," p. 84

1. The common ratio is 3. The next three terms are 81, 243, and 729.

3. The common ratio is -4. The next three terms are 512, -2048, and 8192.

5. The common ratio is 0.5. The next three terms are 4, 2, and 1.

Section 2: Activity 1 (continued)

7. The common ratio is −0.5. The next three terms are 50, −25, and 12.5.

9. The first five terms are 4, 12, 36, 108, and 324.

11. The first five terms are 1000, 200, 40, 8, and 1.6.

13. The first five terms are 2, −8, 32, −128, and 512.

b. Textbook questions 1, 3, 5, 7, and 9 of Investigation 2, "Finding Geometric Means," p. 84

1. 3, 9, 27, 81

3. 6, 12, 24, 48

5. 4, 20, 100 **or** 4, −20, 100

7. −4, −8, −16, −32, −64 **or** −4, 8, −16, 32, −64

9. 216, 36, 6, 1

c. Textbook questions 1 to 3 of Investigation 3, "Applications," p. 85

1. 120% of 10 000 = 12 000
120% of 12 000 = 14 400
120% of 14 400 = 17 280

The painting had a value of $12 000 at the end of the first year, a value of $14 400 at the end of the second year, and a value of $17 280 at the end of the third year.

2. 90% of 80 000 = 72 000
90% of 72 000 = 64 800
90% of 64 800 = 58 320
90% of 58 320 = 52 488

The car had a value of $72 000 at the end of the first year, a value of $64 800 at the end of the second year, a value of $58 320 at the end of the third year, and a value of $52 488 at the end of the fourth year.

3. **a.** 800, 1000 , 1250

The common ratio is 1.25.

In one year, the value of the stamp is expected to be $1000.

b. 800, 1000, 1250, 1562.5

$\times 1.25$

In three years, the value of the stamp is expected to be $1562.50.

16. **a.** Malthus theorized there would be 2 billion in 1823, 4 billion in 1848, 8 billion in 1873, 16 billion in 1898, 32 billion in 1923, 64 billion in 1948, 128 billion in 1973, and 256 billion in 1998.

b. In 1998, Earth's population was about 6 billion.

c. No, he estimated there would be a population of 256 billion in 1998; there was only a population of about 6 billion in 1998.

17. **Textbook question 3 of Investigation 4, "Critical Thinking," p. 85**

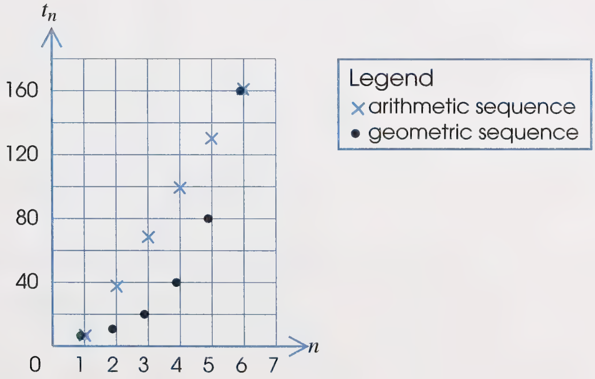
3. **a.** $a = 5$, $n = 6$, and $t_6 = 160$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_6 &= 5 + (6-1)d \\ 160 &= 5 + 5d \\ 155 &= 5d \\ 31 &= d \end{aligned}$$

The sequence is 5, 36 , 67 , 98 , 129 , 160.

b. The sequence is 5, 10 , 20 , 40 , 80 , 160.

c.



Section 2: Activity 1 (continued)

d. The graph of the arithmetic sequence is a straight line; the terms grow at a constant rate. The graph of the geometric sequence is non-linear; the terms grow at an increasing rate.

18. a. Textbook questions 1 to 23 of "Mental Math: Multiplying by Multiples of 5," p. 3

1. $26 \times 5 = 130$
2. $54 \times 5 = 270$
3. $73 \times 5 = 365$
4. $102 \times 5 = 510$
5. $898 \times 5 = 4490$
6. $435 \times 5 = 2175$
7. $5.2 \times 5 = 26$
8. $12.8 \times 5 = 64$
9. $36.4 \times 5 = 182$
10. $56 \times 50 = 2800$
11. $61 \times 50 = 3050$
12. $134 \times 50 = 6700$
13. $256 \times 50 = 12\,800$
14. $3.1 \times 50 = 155$
15. $10.2 \times 50 = 510$
16. Multiplying by 0.5 is the same as dividing by 2.
17. $88 \times 0.5 = 44$
18. $57 \times 0.5 = 28.5$
19. $132 \times 0.5 = 66$
20. $450 \times 0.5 = 225$
21. $9.6 \times 0.5 = 4.8$
22. $14.4 \times 0.5 = 7.2$
23. Multiplying by 5 is the same as multiplying by 10 and then dividing by 2.

$$5 = \frac{10}{2}$$

b. Textbook questions 1 to 26 of "Mental Math: Dividing by Multiples of 5," p. 53

1. $85 \div 5 = 17$
2. $105 \div 5 = 21$
3. $135 \div 5 = 27$
4. $14 \div 5 = 2.8$
5. $39 \div 5 = 7.8$
6. $42 \div 5 = 8.4$
7. $61 \div 5 = 12.2$
8. $123 \div 5 = 24.6$
9. $101 \div 5 = 20.2$
10. $160 \div 50 = 3.2$
11. $210 \div 50 = 4.2$
12. $370 \div 50 = 7.4$
13. $980 \div 50 = 19.6$
14. $135 \div 50 = 2.7$
15. $285 \div 50 = 5.7$
16. $1150 \div 50 = 23$
17. $45 \div 50 = 0.9$
18. $65 \div 50 = 1.3$
19. It is easier because dividing by 0.5 is the same as multiplying by 2.
20. $26 \div 0.5 = 52$
21. $53 \div 0.5 = 106$
22. $118 \div 0.5 = 236$
23. $650 \div 0.5 = 1300$
24. $9.5 \div 0.5 = 19$
25. $3.2 \div 0.5 = 6.4$
26. Dividing by 5 is the same as dividing by 10 and then multiplying by 2.

19. **Step 1:** Consider simpler versions of this problem and look for a pattern.

$$4^1 = 4 \qquad 4^3 = 64 \qquad 4^5 = 1024$$
$$4^2 = 16 \qquad 4^4 = 256 \qquad 4^6 = 4096$$

The simpler version shows that the units digits of the standard form of the powers of 4 have a pattern of 2 repeating digits: 4 and 6.

Step 2: Solve the problem.

The powers with exponents that are odd numbers will have a units digit of 4. The powers with exponents that are even numbers will have a units digit of 6.

Therefore, the standard form of 4^{75} will have a unit digit of 4 because 75 is an odd number.

Section 2: Activity 2

1. $1+3+5+7+9+11+13+15+17+19+21+23+25=13^2$

The sum of the series is 169.

2. **a.** $1+1=2$

$1+1+2=4$

$1+1+2+3=7$

$1+1+2+3+5=12$

$1+1+2+3+5+8=20$

$1+1+2+3+5+8+13=33$

$1+1+2+3+5+8+13+21=54$

b. The sum of the first n Fibonacci terms is equal to $t_{n+2}-1$.

c.
$$\begin{aligned} t_{n+2}-1 &= t_{10+2}-1 \\ &= t_{12}-1 \\ &= 144-1 \\ &= 143 \end{aligned}$$

$t_{11}=89$ and $t_{12}=144$

The sum of the first 10 Fibonacci terms is 143.

Section 2: Activity 2 (continued)

3.

$$\begin{array}{rrrrrrr} 1+ & 2+ & 3+\cdots & +197 & +198 & +199 \\ 199+ & 198+ & 197+\cdots+ & 3+ & 2+ & 1 \end{array}$$

Each pair adds up to 200. There are 199 pairs; so, the rows, together, add up to $199 \times 200 = 39\,800$. Because you only need the sum of one row, divide 39 800 by 2. Therefore, the sum of the series is 19 900.

4. **a.** **Textbook questions 5, 7, and 9 of "Practice," p. 82**

5. $a = 2$, $d = 2$, and $n = 10$

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{10} &= \frac{10}{2}[2(2) + (10-1)2] \\ &= 5(4 + 9 \times 2) \\ &= 5(4 + 18) \\ &= 5 \times 22 \\ &= 110 \end{aligned}$$

7. $a = -2$, $d = 6$, and $n = 30$

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{30} &= \frac{30}{2}[2(-2) + (30-1)6] \\ &= 15(-4 + 29 \times 6) \\ &= 15(-4 + 174) \\ &= 15 \times 170 \\ &= 2550 \end{aligned}$$

9. $a = 80$, $d = -4$, and $n = 15$

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{15} &= \frac{15}{2}[2(80) + (15-1)(-4)] \\ &= \frac{15}{2}[160 + 14(-4)] \\ &= \frac{15}{2}(160 - 56) \\ &= \frac{15}{2} \times 104 \\ &= 15 \times 52 \\ &= 780 \end{aligned}$$

b. Textbook questions 23 and 27 of "Applications and Problem Solving," p. 83

23. $a = 30$, $d = 1$, and $n = 20$

$$\begin{aligned} S_n &= \tfrac{n}{2}[2a + (n-1)d] \\ S_{20} &= \tfrac{20}{2}[2(30) + (20-1)1] \\ &= 10(60 + 19 \times 1) \\ &= 10(60 + 19) \\ &= 10 \times 79 \\ &= 790 \end{aligned}$$

There are 790 seats in the theatre.

27. $a = 4$, $d = 4$, and $n = 12$

$$\begin{aligned} S_n &= \tfrac{n}{2}[2a + (n-1)d] \\ S_{12} &= \tfrac{12}{2}[2(4) + (12-1)4] \\ &= 6(8 + 11 \times 4) \\ &= 6(8 + 44) \\ &= 6 \times 52 \\ &= 312 \end{aligned}$$

There are 312 boxes in the display.

5. a. Textbook questions a. to c. of "Explore: Solve a Simpler Problem," p. 80

a. $1 + 2 + 3 + 4 + 5 = 15$

b. $\frac{15}{5} = 3$

The average is 3.

c. There are 5 terms.

b. Textbook questions 1 to 5 of "Inquire," p. 80

1. The sum of the series is equal to the product of the average and the number of terms.

2. $\frac{1+5}{2} = \frac{6}{2} = 3$

The average is 3.

3. The average of the first and last terms is the same as the average of all the terms.

4. The sum of the series is equal to the product of the number and the average of the first and last terms.

In other words, $S_n = \dfrac{n(a + t_n)}{2}$ or $S_n = \frac{n}{2}(a + t_n)$.

Section 2: Activity 2 (continued)

5.
$$\begin{aligned} S_n &= \frac{n}{2}(a+t_n) \\ &= \frac{15}{2}(1+15) \\ &= \frac{15}{2}\times 16 \\ &= 15\times 8 \\ &= 120 \end{aligned}$$

The sum is 120.

6. a. Textbook questions 10, 12, and 14 of "Practice," p. 83

10.
$$\begin{aligned} t_n &= a+(n-1)d \\ 200 &= 4+(n-1)2 \\ 200 &= 4+2n-2 \\ 200 &= 2+2n \\ 198 &= 2n \\ 99 &= n \end{aligned}$$

Formula 1

$$\begin{aligned} S_n &= \frac{n}{2}(a+t_n) \\ S_{99} &= \frac{99}{2}(4+200) \\ &= \frac{99}{2}(204) \\ &= 99\times 102 \\ &= 10\,098 \end{aligned}$$

Formula 2

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ S_{99} &= \frac{99}{2}[2(4)+(99-1)(2)] \\ &= \frac{99}{2}(8+98\times 2) \\ &= \frac{99}{2}(8+196) \\ &= \frac{99}{2}(204) \\ &= 99\times 102 \\ &= 10\,098 \end{aligned}$$

The sum is 10 098.

12.

$$\begin{aligned}
t_n &= a+(n-1)d \\
-50 &= 100+(n-1)(-10) \\
-50 &= 100-10n+10 \\
-50 &= 110-10n \\
-160 &= -10n \\
16 &= n
\end{aligned}$$

Formula 1

$$\begin{aligned}
S_n &= \frac{n}{2}(a+t_n) \\
S_{16} &= \frac{16}{2}[100+(-50)] \\
&= 8\times 50 \\
&= 400
\end{aligned}$$

Formula 2

$$\begin{aligned}
S_n &= \frac{n}{2}[2a+(n-1)d] \\
S_{16} &= \frac{16}{2}[2(100)+(16-1)(-10)] \\
&= 8[200+15\times(-10)] \\
&= 8(200-150) \\
&= 8\times 50 \\
&= 400
\end{aligned}$$

The sum is 400.

14.

$$\begin{aligned}
t_n &= a+(n-1)d \\
-216 &= 18+(n-1)(-6) \\
-216 &= 18-6n+6 \\
-216 &= 24-6n \\
-240 &= -6n \\
40 &= n
\end{aligned}$$

Formula 1

$$\begin{aligned}
S_n &= \frac{n}{2}(a+t_n) \\
S_{40} &= \frac{40}{2}[18+(-216)] \\
&= 20\times(-198) \\
&= -3960
\end{aligned}$$

Formula 2

$$\begin{aligned}
S_n &= \frac{n}{2}[2a+(n-1)d] \\
S_{40} &= \frac{40}{2}[2(18)+(40-1)(-6)] \\
&= 20[36+39\times(-6)] \\
&= 20[36-234] \\
&= 20\times(-198) \\
&= -3960
\end{aligned}$$

The sum is -3960.

Section 2: Activity 2 (continued)

b. Textbook question 31 of "Applications and Problem Solving," p. 83

31. $d = 6$, $n = 50$, and $S_{50} = 7850$

$$\begin{aligned}
S &= \frac{n}{2}\left[2a+(n-1)d\right]\\
S_{50} &= \frac{50}{2}\left[2a+(50-1)6\right]\\
7850 &= 25(2a+49\times 6)\\
7850 &= 25(2a+294)\\
7850 &= 50a+7350\\
500 &= 50a\\
10 &= a
\end{aligned}$$

$$\begin{aligned}
S_n &= \frac{n}{2}\left(a+t_n\right)\\
7850 &= \frac{50}{2}\left(10+t_n\right)\\
7850 &= 25\left(10+t_n\right)\\
7850 &= 250+25t_n\\
7600 &= 25t_n\\
304 &= t_n
\end{aligned}$$

The first term is 10, and the last term is 304.

7. Textbook questions 6 and 8 of "Practice," p. 82

6. Step 1: Find the general term.

$a = 10$ and $d = 5$

$$\begin{aligned}
t_n &= a+(n-1)d\\
&= 10+(n-1)5
\end{aligned}$$

Note: You do not need to simplify the expression further.

Step 2: Ensure the calculator is in the sequence (Seq) function mode.

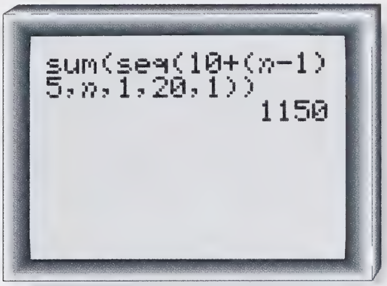
Step 3: Press 2nd [LIST], select the MATH menu, and press 5 to select "sum(."

Step 4: Press 2nd [LIST], select the OPS menu, and press 5 to select "seq(."

Then press the following key sequence to list the general term, the variable, the first term number, the last term number, and the desired increments:

1 0 + (X,T,Θ,n − 1) 5 ,
X,T,Θ,n , 1 , 2 0 , 1)

Press) to close the parentheses for “sum(.” Then press ENTER.



```
sum(seq(10+(n-1)
5,n,1,20,1))
            1150
```

The sum is 1150.

8. Step 1: Find the general term.

Note: You do not need to simplify the expression further.

$a = 40$ and $d = -2$

$$\begin{aligned} t_n &= a+(n-1)d \\ &= 40+(n-1)(-2) \end{aligned}$$

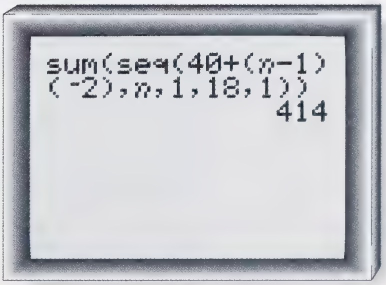
Step 2: Ensure the calculator is in sequence (Seq) function mode.

Step 3: Press 2nd [LIST], select the MATH menu, and press 5 to select “sum(.”

Step 4: Press 2nd [LIST], select the OPS menu, and press 5 to select “seq(.”

Then press the following key sequence:

4 0 + (X,T,Θ,*n* − 1)
((−) 2) , X,T,Θ,*n* ,
1 , 1 8 , 1)



```
sum(seq(40+(n-1)
(-2),n,1,18,1))
             414
```

Press) to close the parentheses for “sum(.”

Press ENTER.

The sum is 414.

Section 2: Activity 2 (continued)

8. Textbook questions 1 and 5 of "Applications and Problem Solving," p. 61

1. **Step 1:** Tia owns the turtle.

	Teacher	Lawyer	Carpenter	Police Officer	Dog	Cat	Parrot	Turtle
Tia					✗	✗	✗	✓
Jane								✗
Fran								✗
Marta								✗
Dog								
Cat								
Parrot								
Turtle								

Step 2: Fran and the carpenter play baseball.

Therefore, Fran is not the carpenter.

	Teacher	Lawyer	Carpenter	Police Officer	Dog	Cat	Parrot	Turtle
Tia					✗	✗	✗	✓
Jane								✗
Fran			✗					✗
Marta								✗
Dog								
Cat								
Parrot								
Turtle								

Step 3: The dog owner and the teacher do not play baseball.

Therefore, Fran isn't the dog owner, Fran isn't the teacher, and the teacher and the carpenter aren't dog owners.

	Teacher	Lawyer	Carpenter	Police Officer	Dog	Cat	Parrot	Turtle
Tia					✗	✗	✗	✔
Jane								✗
Fran			✗		✗			✗
Marta								✗
Dog	✗		✗					
Cat								
Parrot								
Turtle								

Step 4: Tia is not the teacher or the police officer.

Therefore, the teacher doesn't own the turtle and the police officer doesn't own the turtle.

	Teacher	Lawyer	Carpenter	Police Officer	Dog	Cat	Parrot	Turtle
Tia	✗			✗	✗	✗	✗	✔
Jane								✗
Fran	✗		✗		✗			✗
Marta								✗
Dog	✗		✗					
Cat								
Parrot								
Turtle	✗			✗				

Section 2: Activity 2 (continued)

Step 5: Marta sat across from the parrot owner and the dog owner at lunch.

Therefore, Marta isn't the parrot owner or the dog owner. So, Marta must be the cat owner, Jane must be the dog owner, and Fran must be the parrot owner.

	Teacher	Lawyer	Carpenter	Police Officer	Dog	Cat	Parrot	Turtle
Tia	✗			✗	✗	✗	✗	✔
Jane	✗		✗		✔	✗	✗	✗
Fran	✗		✗		✗	✗	✔	✗
Marta	✔	✗	✗	✗	✗	✔	✗	✗
Dog	✗		✗					
Cat	✔	✗	✗	✗				
Parrot	✗							
Turtle	✗			✗				

Step 6: The lawyer doesn't play baseball.

Therefore, Fran can't be the lawyer. So, Fran must be the police officer.

	Teacher	Lawyer	Carpenter	Police Officer	Dog	Cat	Parrot	Turtle
Tia	✗	✗	✔	✗	✗	✗	✗	✔
Jane	✗	✔	✗	✗	✔	✗	✗	✗
Fran	✗	✗	✗	✔	✗	✗	✔	✗
Marta	✔	✗	✗	✗	✗	✔	✗	✗
Dog	✗	✔	✗	✗				
Cat	✔	✗	✗	✗				
Parrot	✗	✗	✗	✔				
Turtle	✗	✗	✔	✗				

Tia is a carpenter and has a pet turtle; Jane is a lawyer and has a pet dog; Fran is a police officer and has a pet parrot; and Marta is a teacher and has a pet cat.

5. **Step 1:** Alonzo did not play country.

	Alonzo	Bonnie	Cari	Deborah	Eric	Jazz	Rock	Classical	Country	R & B
Lennon										
Schultz										
Jones										
Falco										
Zimmer										
Jazz										
Rock										
Classical										
Country	✗									
R & B										

Step 2: Bonnie and Falco had the same teacher.

	Alonzo	Bonnie	Cari	Deborah	Eric	Jazz	Rock	Classical	Country	R & B
Lennon										
Schultz										
Jones										
Falco		✗								
Zimmer										
Jazz										
Rock										
Classical										
Country	✗									
R & B										

Section 2: Activity 2 (continued)

Step 3: The program consisted of Schultz, followed by Bonnie, then the classical guitarist, followed by Lennon, and finally the country guitarist.

	Alonzo	Bonnie	Cari	Deborah	Eric	Jazz	Rock	Classical	Country	R & B
Lennon		✗						✗	✗	
Schultz		✗						✗	✗	
Jones										
Falco		✗								
Zimmer										
Jazz										
Rock										
Classical		✗								
Country	✗	✗								
R & B										

Step 4: After the recital, the guitarists lined up on the stage. From left to right, they were the man who played jazz, Mr. Jones, Deborah, the one who played rhythm and blues, and Zimmer.

	Alonzo	Bonnie	Cari	Deborah	Eric	Jazz	Rock	Classical	Country	R & B
Lennon		✗					✗	✗	✗	
Schultz		✗					✗	✗	✗	
Jones		✗	✗	✗		✗	✗			✗
Falco		✗					✗			
Zimmer	✗	✓	✗	✗	✗	✗	✓	✗	✗	✗
Jazz		✗	✗	✗						
Rock	✗	✓	✗	✗	✗					
Classical		✗	✗							
Country	✗	✗	✗							
R & B	✗	✗	✓	✗	✗					

Step 5: After the recital, Alonzo, Cari, and Schultz took a cab home.

	Alonzo	Bonnie	Cari	Deborah	Eric	Jazz	Rock	Classical	Country	R & B
Lennon	✗	✗	✓	✗	✗	✗	✗	✗	✗	✓
Schultz	✗	✗	✗	✗	✓	✓	✗	✗	✗	✗
Jones	✓	✗	✗	✗	✗	✗	✗	✓	✗	✗
Falco	✗	✗	✗	✓	✗	✗	✗	✗	✓	✗
Zimmer	✗	✓	✗	✗	✗	✗	✓	✗	✗	✗
Jazz	✗	✗	✗	✗	✓					
Rock	✗	✓	✗	✗	✗					
Classical	✓	✗	✗	✗	✗					
Country	✗	✗	✗	✓	✗					
R & B	✗	✗	✓	✗	✗					

Cari Lennon plays R & B; Eric Schultz plays jazz; Alonzo Jones plays classical; Deborah Falco plays country; and Bonnie Zimmer plays rock music.

9. a.	True	**b.**	True	**c.**	False	**d.**	False	**e.**	True
f.	False	**g.**	True	**h.**	False	**i.**	False	**j.**	False

10. a. Textbook question 2 of Investigation 1, "Precipitation in Canada," p. 77

2. a.

$$\begin{aligned} 36\% \text{ of } 5.5 &= 0.36 \times 5.5 \\ &= 1.98 \end{aligned}$$

About 1.98 trillion tonnes of snow falls on Canada in a year.

b.

$$\begin{aligned} 1.98 \text{ trillion tonnes} &= 1.98 \times 10^{12} \text{ t} \\ &= 1.98 \times 10^{15} \text{ kg} \\ &= 1.98 \times 10^{18} \text{ g} \\ &= 1.98 \times 10^{21} \text{ mg} \end{aligned}$$

$$\begin{aligned} 1.98 \times 10^{21} \div 1 \times 10^{24} &= 1.98 \times 10^{-3} \\ &= 0.00198 \\ &\doteq 0.002 \end{aligned}$$

The average mass of one snowflake is about 0.002 mg.

Section 2: Activity 2 (continued)

b. Textbook questions 1 to 3 of Investigation 2, "Greatest Precipitation," p. 77

1. $489.6 \div 24 = 20.4$

About 20.4 mm of rain fell each hour.

2. **a.** 20.4, 40.8, 61.2, …

b. Both a and d equal 20.4 because this is the hourly rainfall.

c.
$$\begin{aligned} t_n &= a + (n-1)d \\ t_{18} &= 20.4 + (18-1)20.4 \\ &= 20.4 + 17 \times 20.4 \\ &= 20.4 + 346.8 \\ &= 367.2 \end{aligned}$$

3. **a.** 0, 20.4, 40.8, …

c.
$$\begin{aligned} t_n &= a + (n-1)d \\ t_{18} &= 0 + (18-1)20.4 \\ &= 0 + 17 \times 20.4 \\ &= 346.8 \end{aligned}$$

Section 2: Follow-up Activities

Extra Help

1. Textbook questions 21 and 22 of "Review," p. 90

21. $a = 3$ and $d = 2$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_8 &= 3 + (8-1)2 \\ &= 3 + 7 \times 2 \\ &= 3 + 14 \\ &= 17 \end{aligned}$$

22. $a = -2$ and $d = -4$

$$\begin{aligned} t_n &= a + (n-1)d \\ t_{25} &= -2 + (25-1)(-4) \\ &= -2 + 24(-4) \\ &= -2 - 96 \\ &= -98 \end{aligned}$$

2. **a.** $a=1$ and $r=3$

$$t_n = ar^{n-1}$$
$$t_8 = 1\times 3^{8-1}$$
$$= 1\times 3^7$$
$$= 1\times 2187$$
$$= 2187$$

b. $a=2$ and $r=3$

$$t_n = ar^{n-1}$$
$$t_{10} = 2\times 3^{10-1}$$
$$= 2\times 3^9$$
$$= 2\times 19\,683$$
$$= 39\,366$$

Enrichment

1. **Textbook question 63 of "Applications and Problem Solving," p. 24**

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$F_2 = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^2$$

$$= \frac{1}{\sqrt{5}}\left[\frac{(1+\sqrt{5})(1+\sqrt{5})}{4}\right] - \frac{1}{\sqrt{5}}\left[\frac{(1-\sqrt{5})(1-\sqrt{5})}{4}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{(1+2\sqrt{5}+5)}{4}\right] - \frac{1}{\sqrt{5}}\left[\frac{(1-2\sqrt{5}+5)}{4}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\frac{(6+2\sqrt{5})}{4}\right] - \frac{1}{\sqrt{5}}\left[\frac{(6-2\sqrt{5})}{4}\right]$$

$$= \frac{6+2\sqrt{5}}{4\sqrt{5}} - \frac{6-2\sqrt{5}}{4\sqrt{5}}$$

$$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4\sqrt{5}}$$

$$= \frac{4\sqrt{5}}{4\sqrt{5}}$$

$$= 1$$

The 2nd Fibonacci number is 1.

Enrichment (continued)

2. Answers will vary.

3. **Numbers**

$$\begin{array}{c} a \\ b \\ a+b \\ a+2b \\ 2a+3b \\ \underline{3a+5b} \\ 5a+8b \\ 8a+13b \end{array}$$

$(8a+13b)-b=8a+12b$

Likewise, if you add the first six numbers, you get
$a+b+(a+b)+(a+2b)+(2a+3b)+(3a+5b)=8a+12b$.

This proves that the sum of the first six numbers is equal to the eighth number minus the second number.

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